

Limits - Review III

1. Given that $f(x) = x^3 - 2x^2 + 5x - 3$, determine $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$.

2. Given that $f(x) = \begin{cases} 4x+2, & \text{if } x > -1 \\ 7-3x, & \text{if } x \leq -1 \end{cases}$, find $\lim_{x \rightarrow -1} f(x)$.

3. Give an example of a function f which is defined at c and for which $\lim_{x \rightarrow c} f(x)$ exists, but $\lim_{x \rightarrow c} f(x) \neq f(c)$.

4. Determine values for c and d so that $f(x)$ is continuous throughout its domain.

$$f(x) = \begin{cases} 3x-5 & x < -2 \\ cx+d & -2 \leq x \leq 3 \\ 5-2x & x > 3 \end{cases}$$

5. Determine c so that $f(x)$ is continuous throughout its domain.

$$f(x) = \begin{cases} c^2x^2 + 2cx - 16 & x \leq 3 \\ 4cx + 19 & x > 3 \end{cases}$$

6. Construct a convincing argument (involving the function values for two CONSECUTIVE INTEGER VALUES for x) using the Intermediate Value Theorem (but NOT finding an explicit value for x and NOT just drawing a graph) that if $f(x) = x^4 - 8x^2 + 2x + 5$ then there must be a value for x so that $f(x) = -5$.

7. Use the ϵ - δ definition for continuity to explain whether $f(x) = \begin{cases} 3x-9 & x \leq 2 \\ 9-x & x > 2 \end{cases}$ is continuous at $x = 2$. If it is not continuous, explain which of the condition(s) fail.