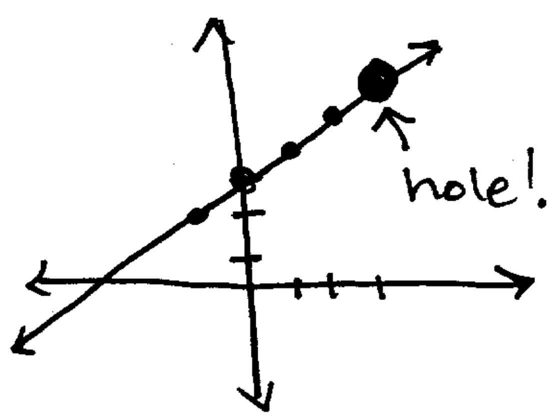
1) $f(x) = \frac{x^2-9}{x-3}$

(a) * I am thinking that there is either a vertical asymptote at $x=3$ or a hole because $x-3$ is in the denominator. So, I am going to factor!

$f(x) = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$ because I can cross out a factor in the numerator and denominator I know there is a HOLE! And, that the graph will look like what's left... $f(x) = x+3$



(b) $\lim_{x \to 3} f(x)$ Even though there is a hole, the limit still exists because both sides of the graph are <u>approaching</u> $y = 6$

$\lim_{x \to 3} \frac{x^2-9}{x-3} = 6$

put 3 in the middle!

#1 (c)

x	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$	5.9	5.99	5.999	ERR	6.001	6.01	6.1

$\longrightarrow$ $\longleftarrow$

approaching 6!

(d) $\lim\limits_{x\to 3} \dfrac{x^2-9}{x-3} = \lim\limits_{x\to 3} \dfrac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} \quad \lim\limits_{x\to 3} x+3 = \boxed{6}$

(e) The graph is a rational function, but has common factors. when the common factors are reduced, a new function $f(x) = x+3$ is left. So, the graph will look just like $f(x) = x+3$ EXCEPT there is a hole at $x=3$ $\to (3,6)$ because the original function is not defined there.

#2 $\lim\limits_{x\to 2} f(x) = \boxed{-3}$ The graph is continuous and approaches -3 from both left & right.

#3 $\lim\limits_{x\to 3} \dfrac{2x+1}{x-3} = \boxed{\text{DNE}}$ There is a vertical asymptote... therefore the limit does not exist (shoots up/down to $\pm\infty$)

#4 $\lim\limits_{x\to 2} \dfrac{x-2}{x^2-2x} = \boxed{\tfrac{1}{2}}$ Doesn't matter that there is a hole! limit still exists!
INTENDED destination

#5 $\lim\limits_{x\to 0} f(x) = \boxed{DNE}$ $\lim\limits_{x\to 0^+} = 1$ $\lim\limits_{x\to 0^-} = 0$

NOT the same!
"JUMP" in graph!

#6 $\lim\limits_{x\to 3} (2x^2-4) \Rightarrow$ DIRECT SUBSTITUTION

$2(3)^2-4 = \boxed{14}$

#7 $\lim\limits_{x\to -1} \dfrac{x^2+3x+2}{x^2+1}$!!! $\dfrac{(-1)^2+3(-1)+2}{(-1)^2+1} = \dfrac{0}{2} = \boxed{0}$

#8 $\lim\limits_{x\to \pi/2} \dfrac{\sin x}{x} \Rightarrow \dfrac{\sin \pi/2}{\pi/2} \Rightarrow$!!! 

$= \dfrac{1}{\pi/2} = \boxed{2/\pi}$

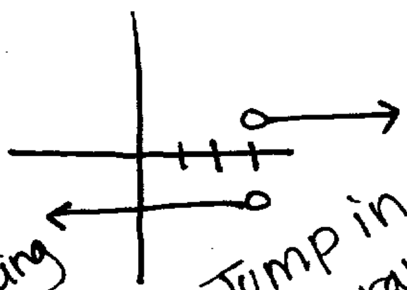
#9 $\lim\limits_{x\to 2} \dfrac{x-2}{x^2-4}$ DIRECT SUB $\Rightarrow \dfrac{2-2}{2^2-4} = \dfrac{0}{0}$ more work!

$\lim\limits_{x\to 2} \dfrac{\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} \Rightarrow \dfrac{1}{2+2} = \boxed{\dfrac{1}{4}}$

#10 $\lim\limits_{x\to -1} \dfrac{x^2-5x-6}{x+1} = \dfrac{(-1)^2-5(-1)-6}{-1+1}$ $\dfrac{1+5-6}{0}$ $\dfrac{0}{0}$ More work!

$\lim\limits_{x\to -1} \dfrac{\cancel{(x+1)}(x-6)}{\cancel{(x+1)}}$ $-1-6 = \boxed{-7}$

#11 $\lim\limits_{x \to 3} \frac{x-3}{|x-3|} = \frac{0}{0}$ Graph these!



limits not approaching same y-values!

Jump in the graph!

x	y
1	-1
2	-1
3	DNE
4	1
5	1

$\boxed{\text{DNE}}$

#12 $\lim\limits_{x \to 0} \frac{\sqrt{x+9}-3}{x}$ $\quad \frac{\sqrt{9}-3}{0} = \frac{0}{0}$ I see $\sqrt{\ }$'s! Rationalize!

$$\lim_{x \to 0} \frac{\sqrt{x+9}-3}{x} \cdot \frac{\sqrt{x+9}+3}{\sqrt{x+9}+3} \Rightarrow \lim_{x \to 0} \frac{x+\cancel{9}-\cancel{9}}{x(\sqrt{x+9}+3)}$$

$$\lim_{x \to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{x+9}+3)} = \frac{1}{\sqrt{0+9}+3} = \boxed{\frac{1}{6}}$$

#13 $\lim\limits_{x \to 6} \frac{1}{(x-6)^2} = \frac{1}{0} =$ Vertical asymptote! $\boxed{\text{DNE}}$

approaching $+\infty$ b/c bottom & top always +

#14 $\lim\limits_{x \to 2^-} \frac{1}{x-2} = \frac{1}{0} =$ vertical asymptote!

coming from the left $\Rightarrow$ negative on bottom
positive on top $\therefore$ approaches $-\infty$

#15 $f(x) = \begin{cases} x^2+1 & x \le 0 \\ 2x-3 & x > 0 \end{cases}$

This is a piece-wise function. This means that it will behave like $y=x^2+1$ for awhile, then change it's mind & start behaving like $y=2x-3$.

The important part is that they connect! So, I am looking for y-values approaching $x=1$ from left & right (checking to see if they are equal)

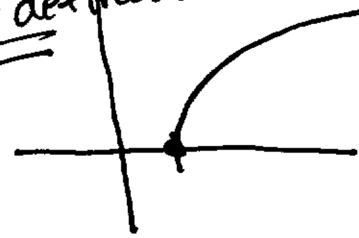
a) $\lim\limits_{x \to 0^-} f(x) =$ plug 0 into top equation

$0^2+1 = \boxed{1}$

b) $\lim\limits_{x \to 0^+} f(x) = 2(0)-3 = \boxed{-3}$

c) $\lim\limits_{x \to 0} f(x) = \boxed{DNE}$ Because they are not the same!

#16 a) $\lim\limits_{x \to 1^-} \sqrt{x-1}$ Function Not defined for $x<1$! No ANSWER



b) $\lim\limits_{x \to 1^+} \sqrt{x-1} = 0$

c) $\lim\limits_{x \to 1} \sqrt{x-1} = 0$ because the function is only defined from right!

17) Find all v.a.'s

$$f(x) = \frac{x^2-x-2}{x^2+x+6}$$

v.a.'s come from where you have zero in the denominator.

However! if the factor can be ELIMINATED, that is not a v.a., but a HOLE

$$f(x) = \frac{(x-2)(x+1)}{(x+3)(x-2)}$$

Nothing crosses out. ∴ (therefore)

$\boxed{x=-3,\ x=2}$

both $x=-3$ and $x=2$ are v.a.'s

#18 $f(x) = \frac{x^3 . 2x^2}{x-2}$

~~[illegible]~~

$$\frac{x^2\cancel{(x-2)}}{\cancel{(x-2)}}$$

a) $\boxed{\text{Discontinuous at } x=2 \text{ REMOVABLE}}$

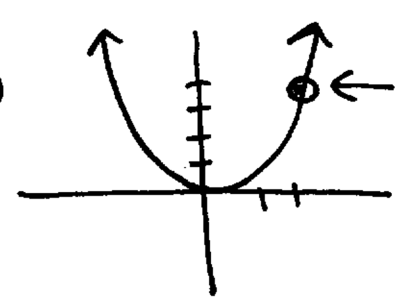
b/c it is a hole & can be factored out

b) find $\lim_{x\to 2} \frac{x^3-2x^2}{x-2} = \frac{0}{0}$

$$\lim_{x\to 2} \frac{x^2\cancel{(x-2)}}{\cancel{(x-2)}} \Rightarrow 2^2 = \boxed{4}$$

c) $g(x) = x^2$

d)



← hole at (2, 4)

#19 Determine c so that f(x) is continuous when

$$f(x) = \begin{cases} x-2 & x \le 5 \\ cx-3 & x > 5 \end{cases}$$

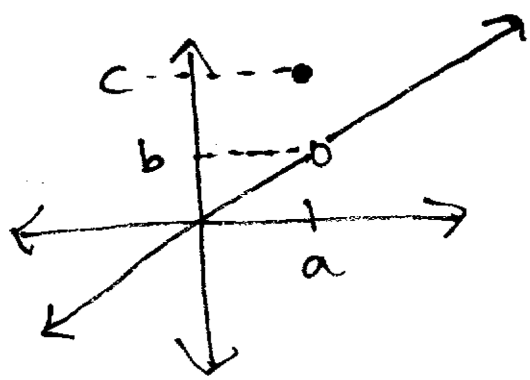
To be continuous ... the function must have a limit!

a) $\lim_{x \to a} f(x)$ must exist!

and, the point associated there must be equal to the limit

b) $\lim_{x \to a} f(x)$ must $= f(a)$

what this means is that the graph has no holes



← NOT CONTINUOUS! The limit exists,

but $f(a) \neq \lim_{x \to a} f(x)$

$f(a) = b$ $\quad \lim_{x \to a} f(x) = c$

So, I need to find what y = when x = 5 on both top and Bottom!

top $5 - 2 = \boxed{3}$

bottom $\boxed{c(5) - 3}$

these must be equal!!

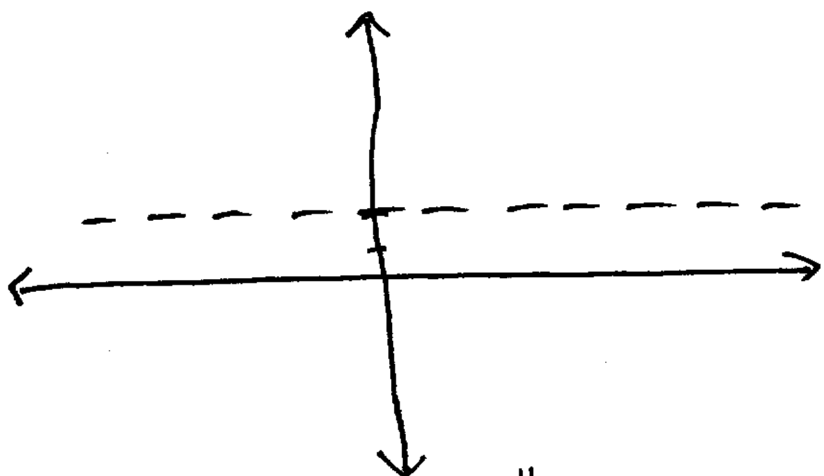
$3 = 5c - 3 \Rightarrow 6 = 5c$

$c = 6/5$

#20 Sketch a graph with these conditions. . .

$\lim_{x \to -\infty} f(x) = 2$ This means that as my x values go off to the left to $-\infty$, my y-values are "topping off at y=2 Horizontal asymptote!

I will create this graph in stages



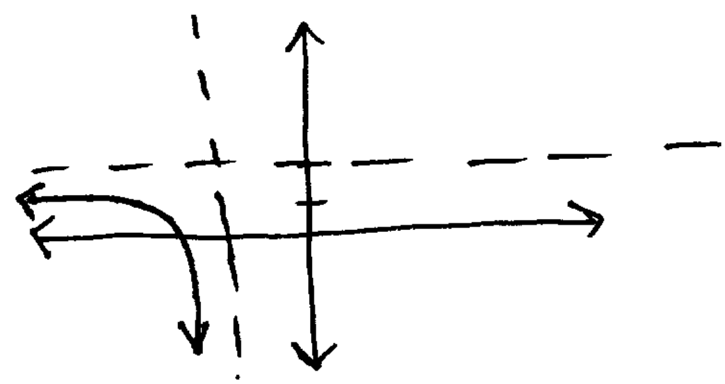
← $\lim_{x \to -\infty} f(x) = 2$

part 2 says, "$\lim_{x \to \infty} f(x) = \infty$" this means my graph will go to $+\infty$ in the y-value direction and NOT flatten out

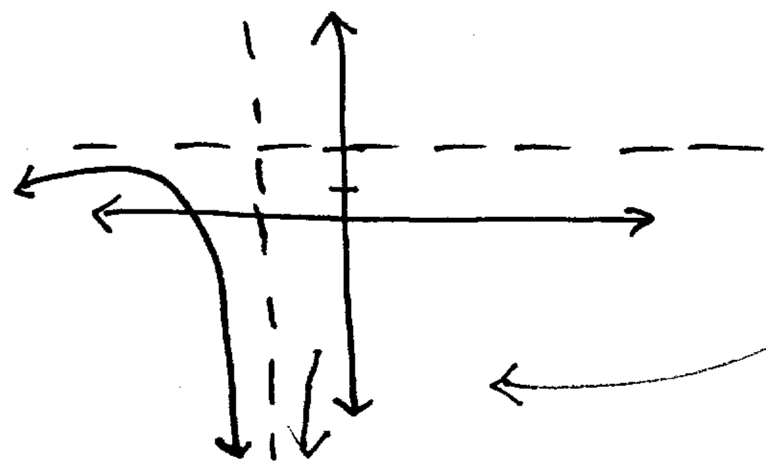
nothing to graph yet!

part 3: $\lim_{x \to -1^-} f(x) = -\infty$ vertical asymptote!!

y=2



part d: $\lim_{x \to -1^+} f(x) = -\infty$

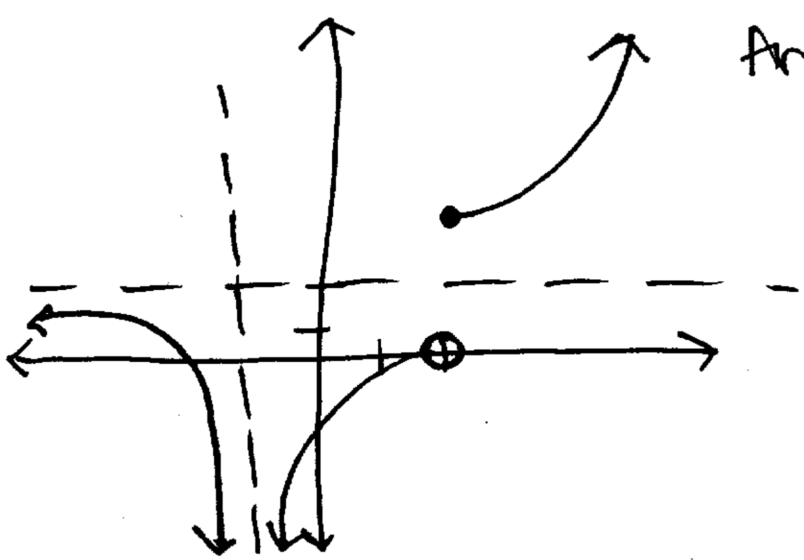


part e: $\lim_{x \to 2^-} f(x) = 0$

part f: $\lim_{x \to 2^+} f(x) = 2$

since they are not approaching the same value – there is a JUMP in the graph at $x = 2$

And, $\lim_{x \to 2} f(x) = \text{DNE}$!



To the left is one example

Answers will vary!