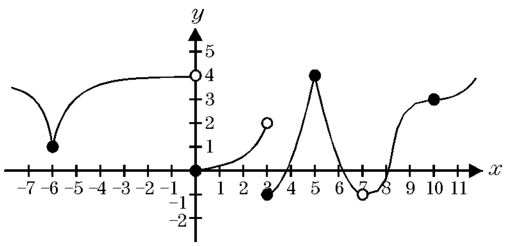
This figure shows the graph of f. Use this figure to answer the following question(s).



1. $\lim_{x\to 7} f$ is

2. $\lim_{x\to 10} f$ is

3. $\lim_{x\to -6} f$ is

4. $\lim_{x\to 0^+} f$ is

5. At which of the following x-values is f continuous? Choose the BEST answer.

    I. -6

    II. 0

    III. 3

    IV. 5

    V. 7

    VI. 10

    a) I, II, and IV
    b) IV and VI
    c) I, IV, and VI
    d) II, III, and V
    e) I and IV

6. At which of the following x-values is f not continuous? Choose the BEST answer.

    I. -6

    II. 0

    III. 3

    IV. 5

    V. 7

    VI. 10

    a) I, II, and IV
    b) IV and VI
    c) I, IV, and VI
    d) II, III, V
    e) I and IV

7. At which of the following x-values does f have a removeable discontinuity? Choose the BEST answer.

    I. -6

    II. 0

    III. 3

    IV. 5

    V. 7

    VI. 10

    a) I, II, and IV
    b) IV and VI
    c) I, IV, and VI
    d) II, III, and V
    e) V only

8. At which of the following x-values does f have a jump discontinuity? Choose the BEST answer.

I. -6

II. 0

III. 3

IV. 5

V. 7

VI. 10

a) I, II, and IV

b) II only

c) I, IV, and VI

d) II, III, and V

e) V only

9. $\lim_{h\to 0} \frac{3(x+h)^2 - 3x^2}{h} =$

10. $\lim_{h\to 9} \frac{x-9}{\sqrt{x}-3} =$

11. $\lim_{h\to 9} \frac{x-9}{\sqrt{x}+3} =$

12. $\lim_{h\to 0} \frac{(x+h)^3 - x^3}{h} =$

13. $\lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} =$

14. $\lim_{x\to -3} \frac{1}{(x+3)^2} =$

15. $\lim_{x\to 3} \frac{x^2 - 8x + 15}{(x-3)^2} =$

16. $\lim_{x\to \frac{1}{2}} \frac{8x^3 - 1}{10x^2 - 7x + 1} =$

17. $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} =$

18. $\lim_{x\to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x-3} =$

19. If $f(x) = \begin{cases} -7 & \text{for } x = 4, \\ 2x+7 & \text{for } x \neq 4 \end{cases}$ then $\lim_{x\to 4} f(x) =$ _________.

20. Find A so that $\lim_{x\to 4} \frac{x^2 + Ax + 20}{x-4}$ exists.

21. If $f(x) = \begin{cases} 12 & \text{for } x \leq -4, \\ -3x & \text{for } -4 < x < 4, \\ -12 & \text{for } x \geq 4 \end{cases}$ then $\lim_{x\to 4} f(x) =$ _________.

22. $\lim_{x\to 0^-} \frac{3}{x}$ is

23. $\lim_{x\to 0^+} \frac{5}{x^3}$ is

24. $\lim_{x\to 1^-} \frac{1}{x-1}$ is

25. $\lim_{x \to 4^-} \frac{2-\sqrt{x}}{\sqrt{4-x}}$ is

26. $\lim_{x \to -\infty} \frac{5x-7}{x}$ is

27. $\lim_{x \to \infty} \frac{3x^4-5x^3+709}{7x^4+9x^2+11}$ is

28. $\lim_{x \to -\infty} \frac{x^2}{(3-x)(3+x)}$ is

29. Given a function defined by $f(x) = \frac{3x-12}{x^2-6x+8}$, for what value(s) of x is the function discontinuous?

30. Which of the following functions are continuous for all real numbers x?

I. $y = -\frac{2}{x-3}$

II. $y = e^x$

III. $y = \csc x$

31. Given a function is defined by $f(x) = \frac{3x-6}{x^2+3x-10}$, for what value(s) of x does the function have one or more vertical asymptotes?

32. For what value(s) of x does the function defined by $f(x) = \frac{x^2+4x-21}{x^2+10x+21}$ have a removeable discontinuity?

33. Let f be defined as follows:

$$f(x) = \begin{cases} \frac{x^2-9}{x-3} & \text{for } x \neq 3, \\ 1 & \text{for } x = 3 \end{cases}$$

Which of the following are true about f?

I. $\lim_{x \to 3} f(x)$ exists

II. $f(3)$ exists

III. $f(x)$ is continuous at $x = 3$

34. $f(x) = \begin{cases} x^2+8 & \text{for } x < 8, \\ a^2x & \text{for } x \geq 8 \end{cases}$

For what value(s) of a is the function continuous?

35. Consider $f(x) = \begin{cases} x^2-5 & \text{for } x < 0, \\ 3 & \text{for } x = 0, \\ x^2+5 & \text{for } x > 0 \end{cases}$

a) $\lim_{x \to 0^+} f(x) =$ _________

b) $\lim_{x \to 0^-} f(x) =$ _________

c) $\lim_{x \to 3} f(x) =$ _________

d) Where is $f(x)$ discontinuous? _________

e) If a function is continuous at $x = a$, does this necessarily mean that $\lim_{x \to a}$ exists?

____

36. Consider $f(x) = \begin{cases} x+c & \text{for } x < 3, \\ cx^2+5 & \text{for } x \geq 3 \end{cases}$

For what value of the constant c is f continuous for all real numbers?

37. By applying the Intermediate Value Theorem choose the interval over which $x^5 = 2x^4 + 11$ will have a solution.

a) $[-2, -1]$ b) $[-1, 0]$ c) $[0, 1]$

d) $[1, 2]$ e) $[2, 3]$

38. By applying the Intermediate Value Theorem choose the interval over which $x^4 = 7x^3 - 5$ will have a solution.

a) $[-2, -1]$ b) $[-1, 0]$ c) $[0, 1]$

d) $[1, 2]$ e) $[2, 3]$

39. Let f be defined by $f(x) = (x^2 - 1)^3$ for all real numbers x. For what values of x is the function decreasing?

40. Let $f(x) = x^2(x + 9)$. Over what interval is the function increasing?

41. Which of the following statements is true of $f(x) = -x^3 + 9x^2 - 24x + 18$?

42. Given that f is continuous and the following information:

Intervals	$x < 4$	$4 < x < 9$	$9 < x$
Sign of f'	$+$	$-$	$+$

a) For what x-value(s) is there a local minimum?

b) For what x-value(s) is there a local maximum?

43. Given that f is continuous and the following information:

Intervals	$x < 0$	$0 < x < 3$	$3 < x$
Sign of f'	$-$	$+$	$-$

a) For what x-value(s) is there a local minimum?

b) For what x-value(s) is there a local maximum?

44. If $f(5) = -4$ and $f'(5) = -4$, then the tangent line approximation at $x = 5$ is ________________.

45. What is the tangent line approximation to $y = 3x^2 - 8x + 2$ for values of x near 3?

46. What is the tangent line approximation to $y = \cos x$ for values of x near $\frac{\pi}{6}$?

47. What is the average rate of change over $2 \le t \le 4$?

t	2	3	4	5	6
$f(t)$	1.8	3.4	4.6	6.4	8.4

48. The position of an object is given by $s = t^2 - 3t + 8$. What is its average velocity for $2 \le t \le 4$?

49. Given the position function $s = t^3 - 2t + 5$, what is the instantaneous rate of change at $t = 3$?

50. The volume of cubic feet of water in a pool is given by the equation $V(t) = 2(t+3)^2$ for $t \geq 0$ and t is in seconds.

  a) What is the average rate of change from $t = 4$ to $t = 6$?

  b) What is the average rate of change over the time interval $[4, 4+t]$?

  c) What is the instantaneous rate of change at $t = 3$?

  d) What is the instantaneous rate of change at any time t?

51. Find all points of inflection: $f(x) = x^3 - 12x$

52. Find the point of inflection of $f(x) = x^3 - 3x^2 - x + 7$.

53. Find all intervals on which the function $y = 8x^3 - 2x^4$ is concave upward.

54. Find all intervals on which the function $y = 6x^3 - x^4$ is concave downward.

55. Find the interval(s) on which the curve $y = 27x^3 + 27x^2 + 9x + 1$ is concave upward or concave downward.

56. If $y = e^{x^2-3x}$, then $y' =$

57. If $y = e^{5/x}$, then $y' =$

58. A curve is defined by $y = e^{\sin 2x}$. Find $\dfrac{dy}{dx}$.

59. Find y' given $y = e^{\sin\sqrt{x}}$.

60. Let $f(x) = x^4 - 2x^3 + 8$. Determine the critical numbers.

61. Find all critical numbers of $f(x) = (9 - x^2)^{3/5}$.

62. Given a function defined by $f(x) = 3x^5 - 5x^3 - 8$, for what value(s) of x is there a relative *maximum*?

63. Given $f(x) = 8x^{1/3} + x^{4/3}$. For what x values does the graph of f have a relative minimum point?

64. Given $f(x) = x^3 + 12x^2 + 18x$ find the absolute maximum value on the closed interval $[-3, 2]$.

65. Find the minimum value of $f(x) = 2x^3 - 3x^2$ on $-2 \leq x \leq 2$.

66. If $f(x) = \dfrac{x^3(x+5)}{x-3}$ then $f'(5)$ is

67. Find an equation of the tangent line to the curve $f(x) = -x^2 + 12$ passing through the point $(4, 0)$.

68. Find the slope of the tangent line to the graph of $f(x) = -3x^2(x^2+2)$ at the point where $x = -1$.

69. If $f(x) = 2x^8 - 15x^7 + 4x^5 + 23x^4 - 7x^2$, then $\dfrac{d^9 f(x)}{dx^9} =$

70. Find the derivative of $y = (x^2 + 2x + 5)^6$.

71. Find the derivative of $y = \sqrt[3]{x^2 + x}$.

72. Find the derivative: $s(t) = \tan \sqrt{t}$

73. Find the derivative of $y = \sin^2 x - \cos^2 x$.

74. Differentiate: $y = \csc^2 \theta + \cot^2 \theta$

75. If $y = \cos(e^x)$, then $\dfrac{dy}{dx} =$

76. Given $y = \sin(x \ln x)$, then $\dfrac{dy}{dx} =$

77. If $x = y + 3y^2 + 4y^3$, then $y' =$

78. Given $2x = xy + y^2$, then $\dfrac{dy}{dx} =$

79. Find y' given $x^2 + y^2 = 2xy$.

80. A projectile starts at time $t = 0$ and moves along the x-axis so that its position at any time $t \geq 0$ is $x(t) = t^4 + 3t^2 + 12t - 20$. What is the velocity of the particle at $t = 1$.

81. The position of a particle at any time t is given by $s = t^3 - 15t^2 + 48t - 10$. What is the velocity after 10 seconds?

82. A particle moves such that its position $x(t) = 20 \sin 5t$ for $t \geq 0$. The acceleration is

83. The position of a particle moving in a straight line at any time t is $x(t) = 2t^2 + 6t + 5$. What is the acceleration of the particle at $t = 3$?

84. How fast is the area of a square increasing when the side is 3 m in length and growing at a rate of 0.8 m/min?

85. The radius of a circle is increasing at the rate of 3 inches per minute. At what rate is the area increasing when the radius is 8 inches?

86. The radius of a sphere is increasing at a constant rate of 0.04 cm per second. At the time when the radius of the sphere is 15 cm, what is the rate of increase of the volume?

Note: The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.

87. A balloon rises vertically at the rate of 8 ft/s. A person on the ground 40 ft away from the spot below the rising balloon watches the balloon ascend; at what rate is the distance between balloon and observer changing when the balloon is 30 ft above ground?

88. Consider the integral $\displaystyle\int \frac{1}{x}$ from $x = 1$ to $x = 4$. Using a Riemann sum with 6 sub-intervals calculate the area under the curve, and above the x-axis, using left endpoints. Answer to 3 decimal places.

89. Integrate: $\int (3x^3 - 2x^2 + 5)\,dx$

90. Integrate: $\int \sqrt[3]{t}\,dt$

91. Evaluate: $\int_0^2 \sqrt{8x^6}\,dx$

92. Evaluate: $\int_1^3 (4x^3 - 3x)\,dx$

93. Evaluate: $\int_0^3 2x(x^3 - 2x^2 + 5)\,dx$

94. Evaluate: $\int_0^2 x^2(x^3 - 5)^2\,dx$

95. Integrate: $\int \cos 4x\,dx$

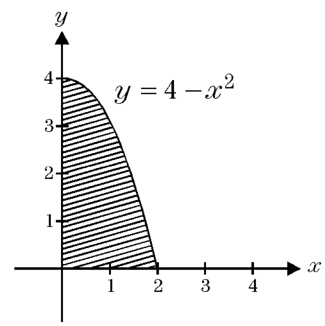
96. Integrate: $\int \sin 3x\,dx$

97. Integrate: $\int 3\csc x \cot x\,dx$

98. On the planet Mathematica the population in the year 2000 was about 8 billion. If the population is growing according to $P(t) = 8e^{0.021t}$ then which definite integral gives the population for the 10-year period starting from the year 2000. Assume $t = 0$ at the beginning of the year 2000.

99. $\int_{\frac{\pi}{2}}^{x} \cos t\,dt = ?$

100. Which of the following definite integrals represents the area of the shaded region?



a) $\int_0^2 (4 - x^2)$

b) $\int_4^2 (4 - x^2)\,dx$

c) $\int_0^2 (4 - x^2)\,dx$

d) $\int_2^4 (4 - x^2)\,dx$

e) $\int_0^4 (4 - x^2)\,dx$

101. Write the definite integral that represents the area of the region enclosed by $y = 4x - x^2$ and the x-axis.

102. A small particle moving in a straight line at any time t with velocity is $v(t) = 2e^t$. How far does the particle move from $t = 0$ to $t = 3$?

103. An object moves in a straight line with velocity $v(t) = 24t - 6t^2$.

a) How far does it travel in the first 4 seconds?

b) What is the *total* distance travelled by the object in the first 5 seconds?

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1.
Answer: -1

2.
Answer: 3

3.
Answer: 1

4.
Answer: 0

5.
Answer: c

6.
Answer: d

7.
Answer: e

8.
Answer: b

9.
Answer: $6x$

10.
Answer: 6

11.
Answer: 0

12.
Answer: $3x^2$

13.
Answer: $-\dfrac{2}{x^3}$

14.
Answer: ∞

15.
Answer: undefined

16.
Answer: 2

17.
Answer: $\dfrac{1}{4}$

18.
Answer: $-\dfrac{1}{9}$

19.
Answer: 15

20.
Answer: -9

21.
Answer: -12

22.
Answer: $-\infty$

23.
Answer: ∞

24.
Answer: $-\infty$

25.
Answer: 0

26.
Answer: -5

27.
Answer: $\frac{3}{7}$

28.
Answer: -1

29.
Answer: 2, 4

30.
Answer: II only

31.
Answer: -5 only

32.
Answer: -7 only

33.
Answer: I and II only

34.
Answer: ± 3

35.
Answer: 5, -5, 4, at 0, yes

36.
Answer: $-\frac{1}{4}$

37.
Answer: e

38.
Answer: b

39.
Answer: $(-\infty, 0)$

40.
Answer: $-\infty < x < -6$ and $x > 0$

41.
Answer: f is increasing on $(2, 4)$

42.
Answer: 9, 4

43.
Answer: 0, 3

44.
Answer: $y + 4 = -4(x - 5)$

45.
Answer: $y = 5 + 10(x - 3)$

46.
Answer: $y = \frac{\sqrt{3}}{2} - \frac{1}{2}\left(x - \frac{\pi}{6}\right)$

47.
Answer: 1.4

48.
Answer: 3

49.
Answer: 25

50.
Answer: $32, 28 + 2t, 24, 4(t + 3)$

51.
Answer: $(0, 0)$

52.
Answer: $(1, 4)$

53.
Answer: $(0, 2)$

54.
Answer: $(-\infty, 0)$ and $(3, \infty)$

55.
Answer: CD $(-\infty, -\frac{1}{3})$; CU $(-\frac{1}{3}, \infty)$

56.
Answer: $(2x - 3)e^{x^2 - 3x}$

57.
Answer: $-\frac{5e^{5/x}}{x^2}$

58.
Answer: $2e^{\sin 2x} \cos 2x$

59.
Answer: $\frac{\cos\sqrt{x}}{2\sqrt{x}} e^{\sin\sqrt{x}}$

60.
Answer: $0,\ \frac{3}{2}$

61.
Answer: $-3,\ 0,\ 3$

62.
Answer: -1 only

63.
Answer: -2 only

64.
Answer: 92

65.
Answer: -28

66.
Answer: 125

67.
Answer: $y = -4x + 16$

68.
Answer: 24

69.
Answer: 0

70.
Answer: $6(2x + 2)(x^2 + 2x + 5)^5$

71.
Answer: $\frac{1}{3}(x^2 + x)^{-2/3}(2x + 1)$

72.
Answer: $\frac{\sec^2\sqrt{t}}{2\sqrt{t}}$

73.
Answer: $2\sin 2x$

74.
Answer: $-4\csc^2\theta\cot\theta$

75.
Answer: $-e^x \sin e^x$

76.
Answer: $\cos(x\ln x)\ln x + \cos(x \ln x)$

77.
Answer: $\frac{1}{1 + 6y + 12y^2}$

78.
Answer: $\dfrac{(2-y)}{x+2y}$

79.
Answer: 1

80.
Answer: 22

81.
Answer: 48

82.
Answer: $-500\sin 5t$

83.
Answer: 4

84.
Answer: $4.8\,\mathrm{m}^2/\mathrm{min}$

85.
Answer: $48\pi\,\mathrm{in}^2/\mathrm{min}$

86.
Answer: $36\pi\,\mathrm{cm}^3/\mathrm{s}$

87.
Answer: $4.8\,\mathrm{ft/s}$

88.
Answer: 1.593

89.
Answer: $\frac{3}{4}x^4 - \frac{2}{3}x^3 + 5x + C$

90.
Answer: $\dfrac{3t^{4/3}}{4} + C$

91.
Answer: $8\sqrt{2}$

92.
Answer: 68

93.
Answer: $\frac{306}{5}$

94.
Answer: $\frac{152}{9}$

95.
Answer: $\frac{1}{4}\sin 4x + C$

96.
Answer: $-\frac{1}{3}\cos 3x + C$

97.
Answer: $-3\csc x + C$

98.
Answer: $\int_0^{10} 8e^{0.021t}\,dt$

99.
Answer: $\sin x - 1$

100.
Answer: c

101.
Answer: $\int_0^4 (4x - x^2)\,dx$

102.
Answer: $2(e^3 - 1)$

103.
Answer: 64, 78