AP Calculus
Journal Questions

These Journal Questions (JQ) are designed to develop a deeper understanding of the AP Calculus concepts that are discussed in class. Many of the questions on the AP-level Unit tests and the final AP exam itself are designed to assess your understanding of the connections between the different concepts. These connections cannot be established just by doing calculation-based problems alone. In order to develop a keen understanding of these connections, it is necessary to discuss the concepts verbally and in writing. The more time and effort you put into answering these JQ, the easier it will become for you to solve the actual numeric AP problems. If you do not attempt to develop this understanding, you will have considerable difficulty with the assessments.

It is your responsibility to make sure your answers to the JQ are correct and complete. You can either bring them to me for checking or discuss your answer with a peer who you trust has a good understanding of the concepts.

You are required to answer these JQ in a separate Journal or on loose leaf paper in a section of your binder. I would suggest a separate Journal because then you can use it as a reference/refresher in College, next year!

In order for this process to be useful to you now and in the future, will require considerable time and effort on your part. However, we all know that any worthwhile cause will require time and effort, and anything that requires little or no effort is usually not worth doing!!! You may find it tough while you're doing it but one year from now, you'll look back and be very glad that you did it!!!

Unit 1 - Limits

1. Explain how to evaluate $\lim_{x \to 2} f(x)$ numerically.

2. Explain how to evaluate $\lim_{x \to 2} f(x)$ graphically.

3. Explain the difference in the <u>meanings</u> of $\lim_{x \to 2} f(x)$ and $f(2)$.

4. When will the values of $\lim_{x \to 2} f(x)$ and $f(2)$ be equal and when will their values not be equal?

5. If $f(x)$ is continuous at $x = a$, why is it possible to calculate the value of $\lim_{x \to 2} f(x)$ by simply evaluating $f(a)$?

6. If $f(x)$ has a point discontinuity at $x = c$, how would you modify the definition of $f(x)$ to make it continuous for $x \in \Re$.

7. Given, $f(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3} & ; \ x \neq 3 \\ 10 & ; \ x = 3 \end{cases}$. Modify the definition of $f(x)$ so that it is continuous for $x \in \Re$.

8. State the IVT and explain how it can guarantee the existence of zeros of a function, $f(x)$.

Unit 2 - Differentiation

9. Explain, graphically, how the expression for the instantaneous ROC of $f(x)$ at $x=a$ can be obtained from the expression that gives the slope of a secant line between $x=a$ and another point on $f(x)$.

10. Explain the difference between the usage of the two expressions given below:

    I. $\lim_{b \to a} \frac{f(b)-f(a)}{b-a}$ II. $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

11. List the <u>four</u> interpretations of the expression: $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$.

12. To determine if $f(x)$ is differentiable at $x=a$, why is it necessary to determine if $f(x)$ is differentiable for $x \to a^{+}$ <u>and</u> $x \to a^{-}$?

13. Explain how to determine if a PDF is differentiable at $x=a$.

14. Use examples to explain the statement: *"Continuity does not guarantee Differentiability."*

15. Explain how to find the equation of a tangent line to a curve at a point.

16. Explain how to approximate the rate of change of a function from graphs and tables of values.

17. Discuss the following statement: *"The Power Rule is just a simplified version of the Chain Rule."*

18. Explain how to use the chain rule for differentiating composite functions.

19. Explain why: $\frac{d}{dx}\left(y^{n}\right)=ny^{n-1}\frac{dy}{dx}$. In particular, explain the need for the $\frac{dy}{dx}$ in the answer.

20. Explain the meaning of a *Linear Approximation to* $f(x)$ *at* $x=c$.

21. Explain how to find the derivative of $f^{-1}(x)$ at $x=a$ if you are given $f(x)$.

22. Explain how to sketch the graph of $f'(x)$ from the graph of $f(x)$.

Unit 3 - Applications of Derivatives

23. Explain how to create a *Sign Chart* for a function. Explain why a *Sign Chart* can be considered to be a "summary" of the graph of the function?
24. Explain WHY the first derivative can be used to determine when a function is increasing/decreasing.
25. Explain HOW to determine the monotonicity of $f(x)$, given the equation of $f'(x)$.
26. Explain HOW to determine the monotonicity of $f(x)$, given the graph of $f'(x)$.
27. Explain WHY the first derivative can be used to determine the local max/min of a function.
28. Explain HOW to determine the local max/min of $f(x)$, given the equation of $f'(x)$.
29. Explain HOW to determine the local min/max of $f(x)$, given the graph of $f'(x)$.
30. Discuss the statement: *"If $f(x)$ has a critical value at $x=c$ then $f(x)$ has a local max/min at $x=c$."*
31. Explain WHY the 2nd derivative can be used to determine when a function is CU or CD.
32. Explain HOW to determine the concavity of $f(x)$, given the equation of $f''(x)$.
33. Explain HOW to determine the concavity of $f(x)$, given the graph of $f''(x)$.
34. Explain how the graph of $f'(x)$ can also be used to determine the concavity of $f(x)$.
35. What is the significance of a POI on the graph of $f(x)$? Answer in terms of rate of change of $f(x)$. You may use a real life example (e.g. $f(x)=$a profit function) to help you formulate your answer.
36. How can the location of a POI be determined using $f'(x)$ and also by using $f''(x)$?
37. Explain the difference between the *1st Derivative Test* and the *2nd Derivative Test*. When does the *2nd Derivative Test* "fail"?
38. Discuss the statement: *"The displacement of an object and the distance travelled by an object can both be found by using the exact same calculations."*
39. Explain how to use a GC to calculate local max/min, POI, and intervals of concavity given the equation for $f(x)$.

40. Explain the difference between a *Vertical Asymptote* and a *Horizontal Asymptote* in terms of:

i. limit of $f(x)$

ii. what they tell us about the behaviour of the graph of $f(x)$ and where

iii. how to actually find them

Unit 4 - Antiderivatives & Integration

41. Explain why a *"+C"* is necessary when finding the *General Antiderivative* of a function. How can the value of *C* be obtained?

42. Explain the difference between TOTAL area and NET/SIGNED area.

43. How can the expression, $\int_a^b f(x)dx$ be used to find TOTAL area and NET/SIGNED area?

44. Explain how to approximate area under a curve using: LRAM, RRAM, MRAM, and Trapezoids, given:

    a. definition of $f(x)$

    b. table of values

45. When does each of LRAM, RRAM, MRAM, and Trapezoid methods over-estimate and under-estimate the actual area?

46. Discuss some of the other interpretations of the "area" under $f(x)$, other than just the physical area of the "shape."

47. What does the FTC I tell us about the relationship between the derivative of a function and the accumulated area under the graph of a function?

48. Explain FTC II and show how the result is obtained from FTC I.

49. Explain how to evaluate: $\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)dt$ using FTC I.

50. Explain why: $\int_a^b f(t)dt \neq \int_b^a f(t)dt$

51. Explain <u>how</u> to determine the value of $\int_a^b f(t)dt$ (and what needs to be given to you in the question), if using:

a. area model

b. antiderivatives (2 answers here)

c. technology

d. approximation

52. Given, $g(x)=\int_a^x f(t)\,dt$, explain how to determine the following facts about the accumulated area function $g(x)$ given (i) equation of $f(t)$ and (ii) graph of $f(t)$:

a. increasing/decreasing

b. local max/min and absolute max/min

c. POI

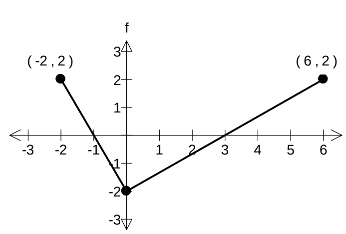
d. intervals of concavity

53. Explain why $\int_a^b f'(t)\,dt = f(b)-f(a)$

54. Explain how $\int_a^b f'(t)\,dt = f(b)-f(a)$ can be used to determine the NET change of a quantity. Use 2 or 3 different "real-world" examples to support your explanation.

55. Explain why the following statement is incorrect: "$\int_a^b v(t)\,dt =$ *final position of an object*." What is the correct method for determining the final position of an object moving with velocity, $v(t)$?

56. Given the graph of $f(t)$ below, sketch the graph of the function, $g(x)=\int_3^x f(t)\,dt$.



57. If you are given the definition of $f(t)$ and the value of the antiderivative of $f(t)$ at $x=b$, explain how to use an integral function to determine the value of the antiderivative of $f(t)$ at $x=a$.

58. $R(t)=$ Rate of the flow of water into a filtering system ($litres/hour$)

$L(t)=$ Rate of the flow of water out of the filtering system ($litres/hour$)

$C(t)=$ Cost of filtering water at any time, *t* ($\$/litre$)

Given the above information, set up an integral to determine:

a. the volume of water (in litres) in the filtering system at any time, *t*, given that the system has 200 litres of water at time, $t=0$.

b. the total cost of filtering the water up to any time *t.*

59. Explain when and how to use *U*-substitution to find the antiderivative of a function.

Units 5 & 6 - Areas, Volumes of Solids, & Differential Equations

60. Explain exactly what the integral, $\int_a^b (expression)\, dx$ actually calculates, including the importance and purpose of the dx.

61. Explain how the integral expression discussed in question 60 is used to determine each of the following:

    a. Area bounded by 2 curves.

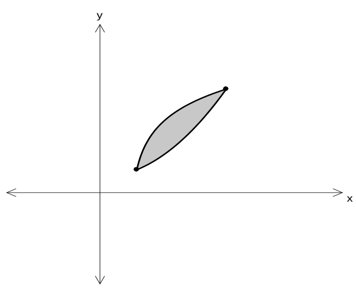
    b. Area bounded by 3 curves.

    c. Volume of a solid by revolution, using washers.

    d. Volume of solid with known cross-sections that has base as the area bounded by 2 functions.

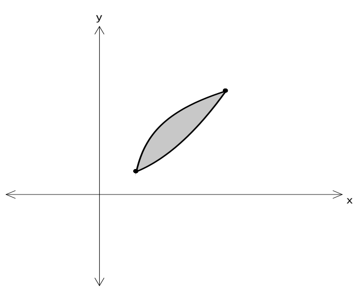
62. Setup an integral to calculate the volume of the solid obtained by rotating the area given below about each of the axis indicated. The top function is *f* and the bottom function is *g*.

    a.



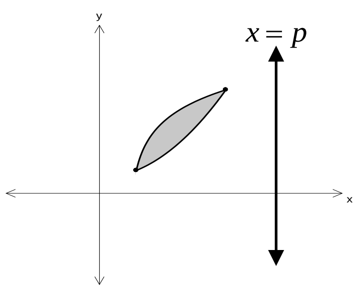
Rotate about the x-axis

    b.

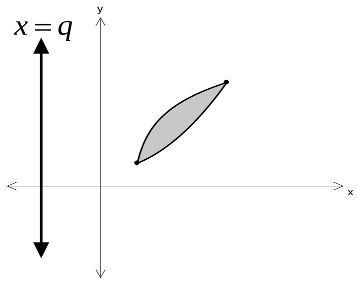


Rotate about the y-axis

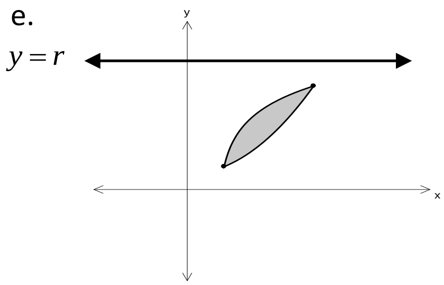
    c.



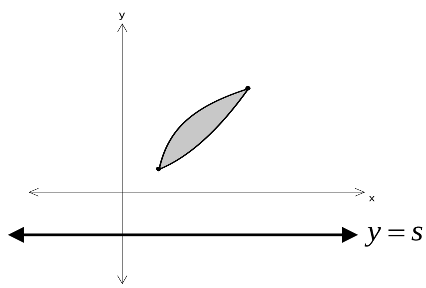
Rotate about $x = p$

d. 

Rotate about $x=q$

e. 

Rotate about $y=r$

f. 

Rotate about $y=s$

63. Explain the difference between *Slope Fields* and *Solution Curves* for a Differential Equation.

64. List the steps for solving a separable DE.