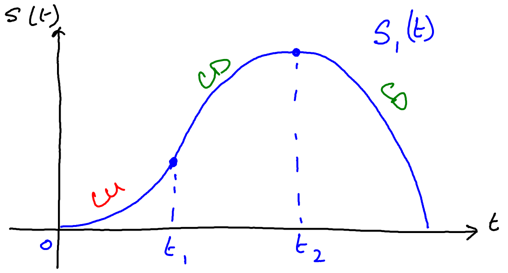
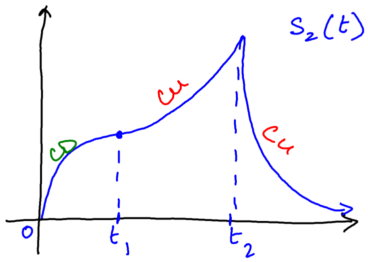
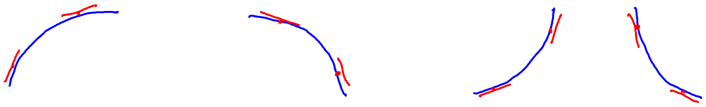
3.3A - Concavity, Points of Inflection, 2nd Derivative Test

January-07-16
12:17 PM



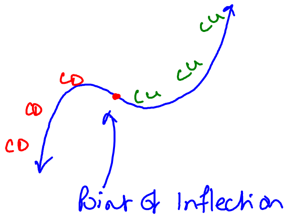


curves are made up of 4 distinct shapes!



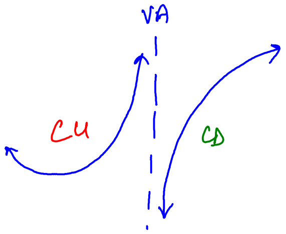
	Increasing CD	Decreasing CD	Increasing CU	decreasing CU
f':	+	−	+	−
f'':	−	−	+	+

Concavity change



⇒ Point at which $f(x)$ changes concavity.

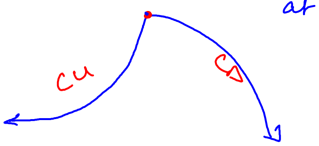
f' is defined
f'' " "



Concavity Changes, but no PoI.

f' u/d
f'' u/d
f u/d

PoI and $f(x)$ is defined at PoI



f' u/d
f'' u/d
f is defined.

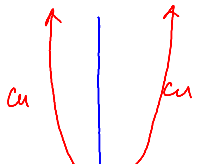
$x=c$ is a PoI if: i) $f(c)$ is defined.

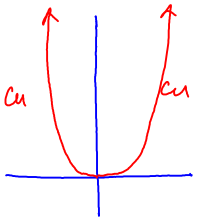
and ii) $f(x)$ changes concavity about $x=c$.

⇒ $f''(x)$ changes sign about $x=c$.

$f(x) = x^4$ $f'(x) = 4x^3$ $f''(x) = 12x^2$ $f''(0) = 0$

But no change in concavity.



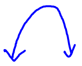


But no change in concavity.

2nd Derivative Test.

If $f'(c) = 0$ ⇒ there is a local max or local min at $x = c$

$\downarrow$ local max: $f''(c) < 0$



CD.

$\downarrow$ local min: $f''(c) > 0$

CU

e.g. $f(x) = e^x(2x - x^2)$ Find all intervals of concavity

$f'(x) = e^x(2-2x) + e^x(2x - x^2)$

$= 2e^x - \cancel{2e^x x} + \cancel{2e^x x} - x^2 e^x$

$f'(x) = 2e^x - x^2 e^x$

$f''(x) = 2e^x - [x^2 \cdot e^x + 2x \cdot e^x]$

$f''(x) = 2e^x - x^2 e^x - 2x e^x$

Solve: $2e^x - x^2 e^x - 2xe^x = 0$

$(e^x)(-x^2 - 2x + 2) = 0$

$-e^x(x^2 + 2x - 2) = 0$

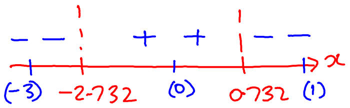
When: i) $e^x = 0$ ⇒ No sol^n.

ii) $x^2 + 2x - 2 = 0$

$$x = \frac{-2 \pm \sqrt{2^2 - (4)(1)(-2)}}{2}$$

$= -2.732, \ 0.732$

$f''(x)$:



$f''(-3) < 0$

$f''(0) > 0$

$f''(1) < 0$

CD on $x = (-\infty, -2.732) \cup (0.732, \infty)$

CU on $x = (-2.732, 0.732)$.

Assign: w/s 3-3 + p. 242: 1-6, 7, 8, 12, 15, 20, 21, 23, 31, 37, 43, 56, 57.