

Definite Integration with u-Substitution - Classwork

When you have to find a definite integral involving *u-substitution*, it is often convenient to determine the limits of integration in terms of the variable u , rather than having to integrate and switch back to x and then substitute. This is called “changing the limits.”

Example 1) $\int_0^2 x(x^2 + 1)^2 dx$ Start off by finding u _____ $x = 2, u =$ _____

$\frac{1}{2} \int_0^2 2x(x^2 + 1)^2 dx$ $du =$ _____ Do we have it? $x = 0, u =$ _____

Now write everything in terms of u and calculate. It is no longer necessary to switch back to x .

Example 2) $\int_0^1 x\sqrt{1-x^2} dx$

Example 3) $\int_0^{\sqrt{5}} \frac{x}{\sqrt{x^2 + 4}} dx$

Example 4) $\int_0^{\pi/4} \sin 2x dx$

Example 5) $\int_0^{\pi/2} \sin x \sqrt{\cos x} dx$

Example 6) $\int_0^{\pi/6} (1 + \sin x \cos x) dx$

Example 7) $\int_{-2}^2 |1 - x^2| dx$

Definite Integration with u-Substitution - Homework

Find the values of the following definite integrals. Verify using your calculator. Some will use u -substitution, others will not.

$$1. \int_{-2}^2 (x^3 - 1) dx$$

$$2. \int_0^4 x(\sqrt{x} - 1) dx$$

$$3. \int_0^{\pi/3} \sin(2x) dx$$

$$4. \int_0^{\pi/12} (1 - \cos 2x) dx$$

$$5. \int_0^1 2x(x^2 + 1)^2 dx$$

$$6. \int_0^3 x\sqrt{9 - x^2} dx$$

$$7. \int_0^5 |x - 4| dx$$

$$8. \int_0^4 |x - \sqrt{x}| dx$$

$$9. \int_2^3 \frac{x}{(x^2 - 3)^2} dx$$

$$10. \int_0^4 \frac{dt}{\sqrt{2t+1}}$$

$$11. \int_0^{\pi/2} \cos^3 t \sin t dt$$

$$12. \int_0^{\sqrt{\pi/2}} t \sin(\pi - t^2) dt$$

$$13. \int_0^{\pi/4} \sqrt{\tan x} \sec^2 x \, dx$$

$$14. \int_0^{\pi/3} \cos x \sqrt{1 - \cos^2 x} \, dx$$

$$15. \int_0^1 x \sqrt{ax^2 + b} \, dx$$

$$16. \int_{\pi^2/4}^{\pi^2} \frac{\sin \sqrt{x}}{\sqrt{x}} \, dx$$

$$17. \int_0^4 |9 - x^2| \, dx$$

$$18. \int_{-4}^4 \frac{1}{x^2} \, dx \text{ (Be careful!)}$$

If $\int_0^2 f(x) \, dx = \frac{11}{3}$ and $\int_0^6 f(x) \, dx = 15$, $f(x)$ is an even function (symmetric to the y -axis), find the following:

$$19. \int_{-2}^0 f(x) \, dx$$

$$20. \int_{-2}^2 f(x) \, dx$$

$$21. \int_0^2 -f(x) \, dx$$

$$22. \int_{-2}^0 3f(x) \, dx$$

$$23. \int_0^2 f(3x) \, dx$$

If $\int_0^2 f(x) \, dx = \frac{11}{3}$ and $\int_0^6 f(x) \, dx = 15$, $f(x)$ is an odd function (symmetric to the origin), find the following:

$$24. \int_{-2}^0 f(x) \, dx$$

$$25. \int_{-2}^2 f(x) \, dx$$

$$26. \int_0^2 -f(x) \, dx$$

$$27. \int_{-2}^0 3f(x) \, dx$$

$$28. \int_{-2}^2 f(3x) \, dx$$