# AP Calculus
## Unit 6 – Differential Equations
### *GA – AP FR Questions*

1. Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

   (a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.

   (b) Find the domain and range of the function $f$ found in part (a).

2. The function $f$ is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point $(x, y)$ on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

   (a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

   (b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \frac{1}{4}$.

   c) Determine whether $f(x)$ has a local max or min at $\left(3, \frac{1}{4}\right)$. Justify your answer.

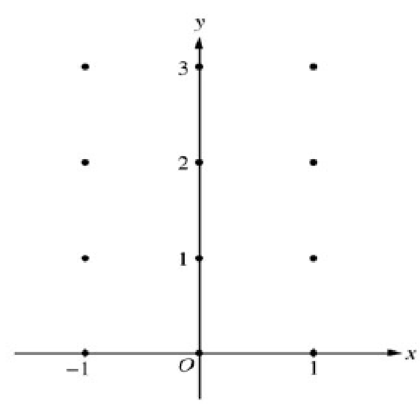
   d) Determine the monotonicity and concavity of $f(x)$ at $(0, 1)$. Justify your answers.

3. Consider the differential equation $\frac{dy}{dx} = x^2(y - 1)$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. (**Note: Use the axes provided in the pink test booklet.**)

   (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the $xy$-plane. Describe all points in the $xy$-plane for which the slopes are positive.

   (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

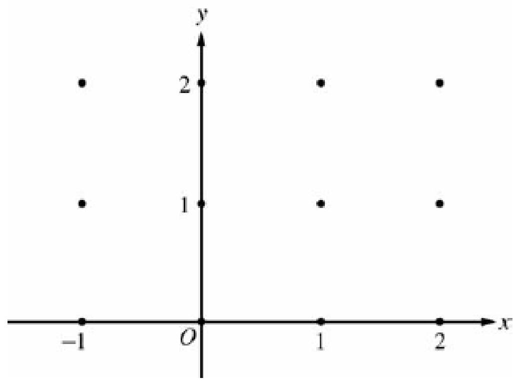



   d) Determine the equation of the line that is tangent to $f(x)$ at the point $(1, 0)$.

   e) Use the tangent line to approximate the value of $f(1.1)$. Is your value an over-estimate or under-estimate of the actual value? Justify your answer.

4. Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
**(Note: Use the axes provided in the test booklet.)**

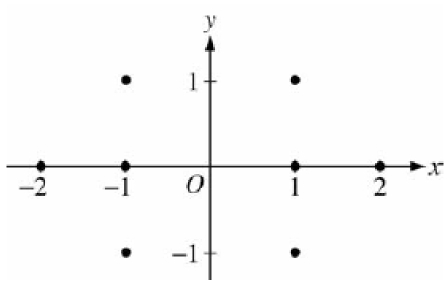



(b) Write an equation for the line tangent to the graph of $f$ at $x = -1$.

(c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

5. Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

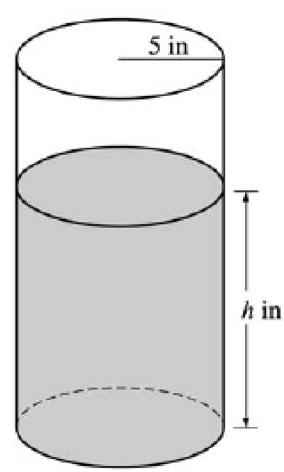
(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.
**(Note: Use the axes provided in the pink exam booklet.)**




(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

c) Determine the value of $f(-1.2)$ to three decimal places, where $f(x)$ is the function that satisfies the initial condition $f(-1) = 1$.

6. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let $h$ be the depth of the coffee in the pot, measured in inches, where $h$ is a function of time $t$, measured in seconds. The volume $V$ of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume $V$ of a cylinder with radius $r$ and height $h$ is $V = \pi r^2 h$.)




(a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

(b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for $h$ as a function of $t$.

(c) At what time $t$ is the coffeepot empty?

**Solutions**

1. a. $y=\frac{1}{2}\ln\left(2x^3+e\right)$ b. $D:\ x>-\left(\frac{1}{2}e\right)^{\frac{1}{3}};\ \ R:\ (-\infty,\infty)$

2. a. $2y^3(6-2x)^2-2y^2;\ -\frac{1}{8}$ b. $y=\frac{1}{x^2-6x+13}$ c. max. d. increasing, CU

3. b. $x\neq 0,\ y>1$ c. $y=1+2e^{\frac{x^3}{3}}$ d. $y=1-x$ e. $-0.1$, over

4. b. $y-2=2(x+1)$ c. $y=\frac{4}{x^2+1}$

5. b. $y=-2x-1,\ x<0$ c. 1.4

6. b. $h=\left(-0.1t+\sqrt{17}\right)^2$ c. $10\sqrt{17}$ sec