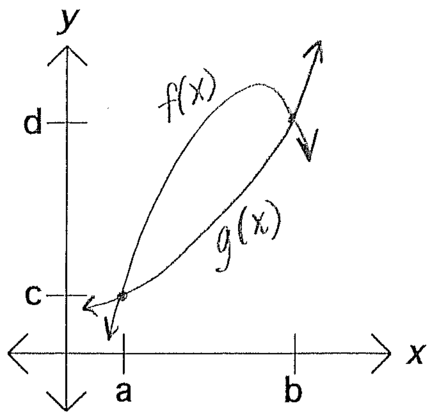
Answer Key

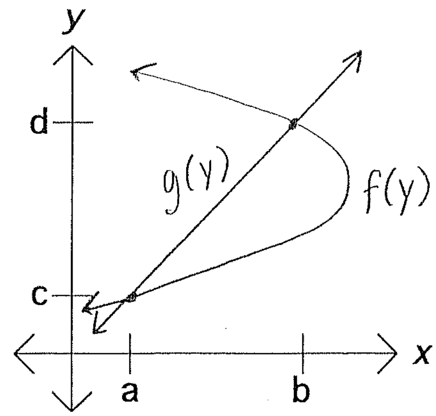
CHAPTER 7 REVIEW: AREA AND VOLUME

AREA BETWEEN TWO CURVES

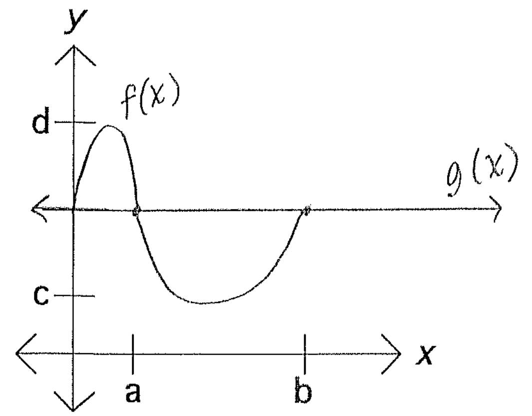
Set up an integral to determine the area bounded by the two curves.



$\int_a^b [f(x) - g(x)]\,dx$



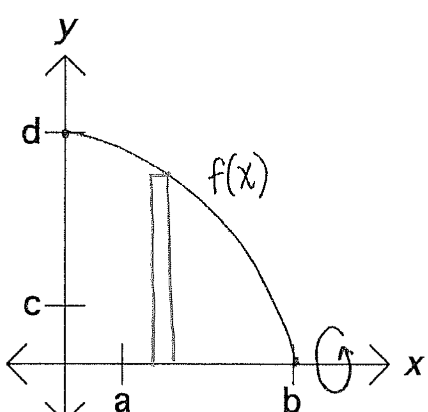
$\int_c^d [f(y) - g(y)]\,dy$



$\int_0^a [f(x) - g(x)]\,dx + \int_a^b [g(x) - f(x)]\,dx$

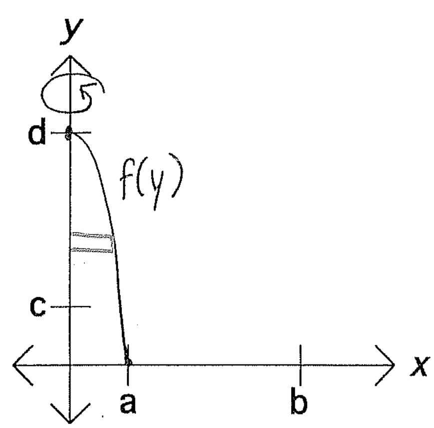
VOLUME OF A SOLID OF REVOLUTION

Set up an integral to determine the volume of the solid. Indicate whether you used the disc, washer or shell method. Draw the representative rectangle. For purposes of this review sheet, if the graphs are labeled f(x) and g(x), then choose a method that allows you to integrate with respect to x. Likewise, if the graphs are labeled f(y) and g(y), choose a method that allows you to integrate with respect to y.



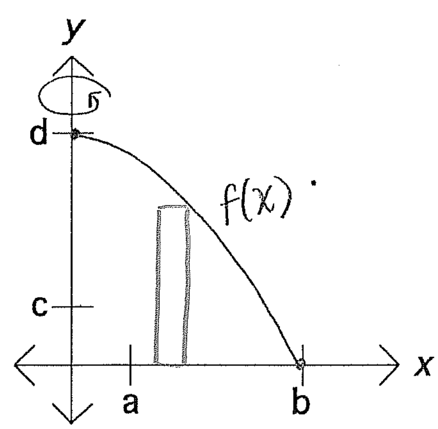
$\pi \int_0^b [f(x)]^2\,dx$

disc



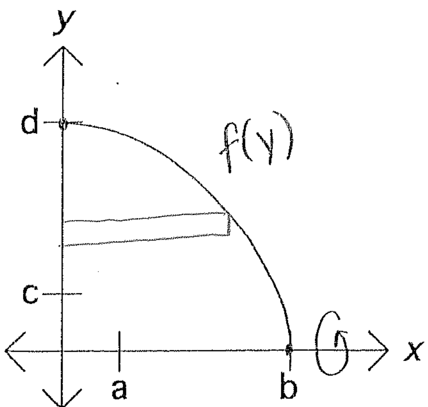
$\pi \int_0^d [f(y)]^2\,dy$

disc



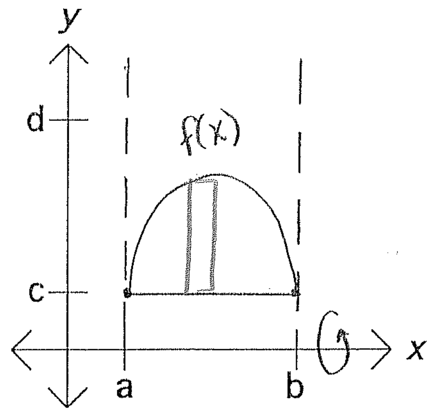
$2\pi \int_0^b x \cdot f(x)\,dx$

shell



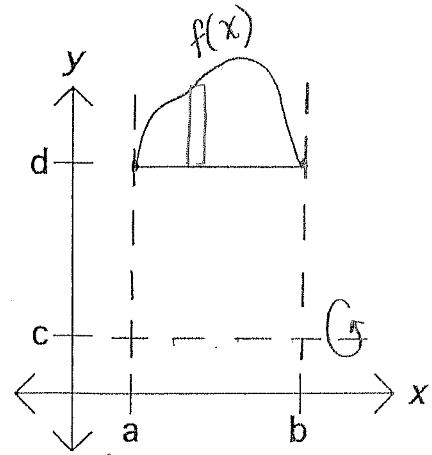
$2\pi\int_0^d y\, f(y)\, dy$

shell



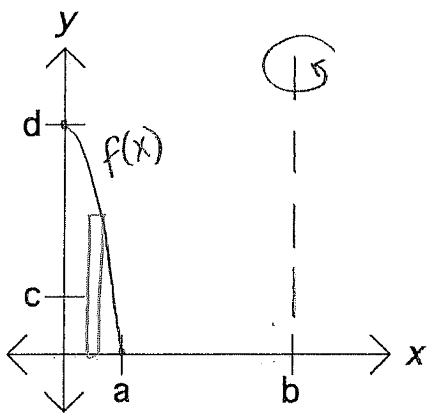
$\pi\int_a^b \left([f(x)]^2 - c^2\right) dx$

washer



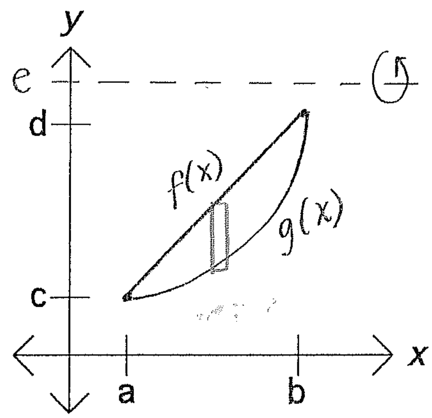
$\pi\int_a^b \left([f(x)-c]^2 - [d-c]^2\right) dx$

washer



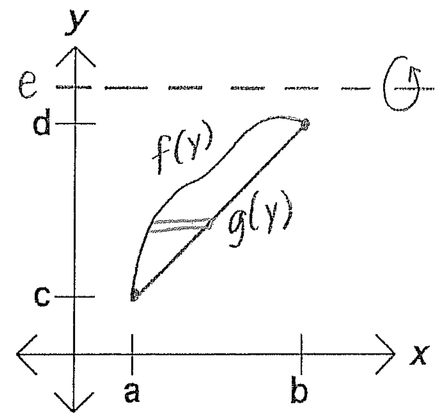
$2\pi\int_0^a (b-x) f(x)\, dx$

shell



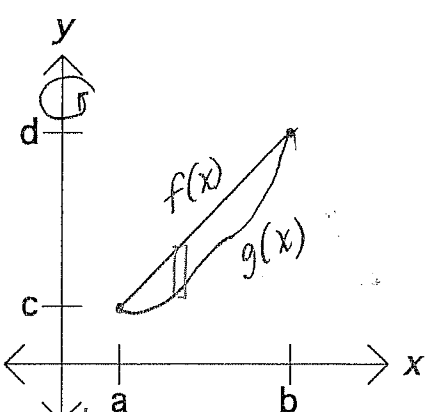
$\pi\int_a^b \left([e-g(x)]^2 - [e-f(x)]^2\right) dx$

washer



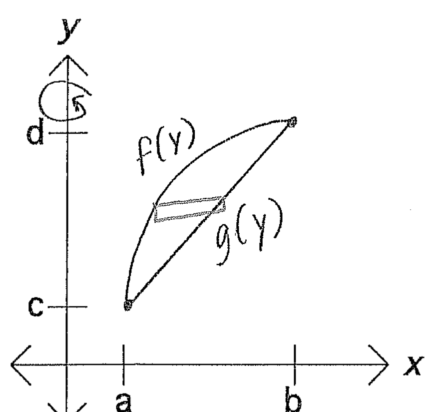
$2\pi\int_c^d (e-y)[g(y)-f(y)]\, dy$

shell



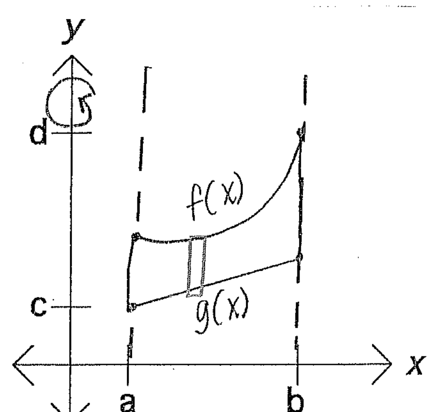
$2\pi\int_a^b x[f(x)-g(x)]\, dx$

shell



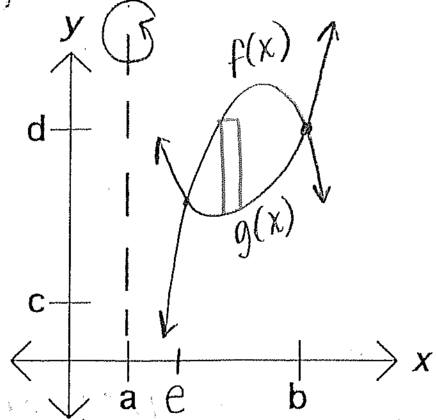
$\pi\int_c^d [g(y)]^2-[f(y)]^2 dy$

washer



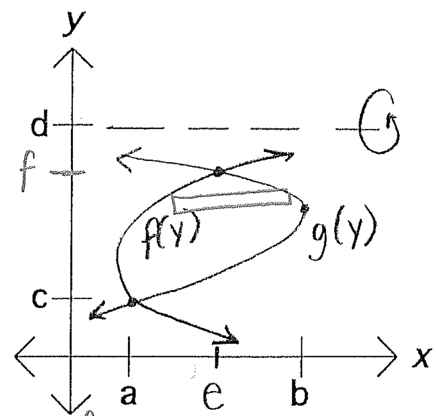
$2\pi\int_a^b x[f(x)-g(x)]\, dx$

shell



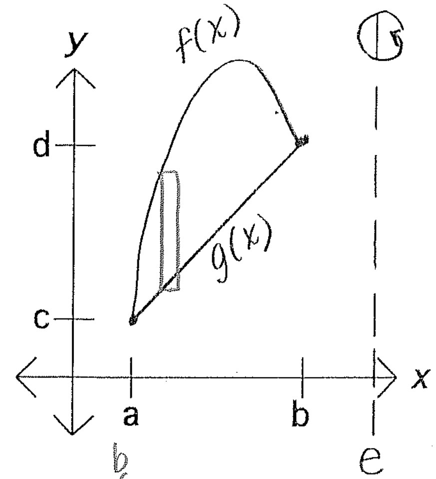
$$2\pi \int_{e}^{b} (x-a)[f(x)-g(x)]\,dx$$

shell



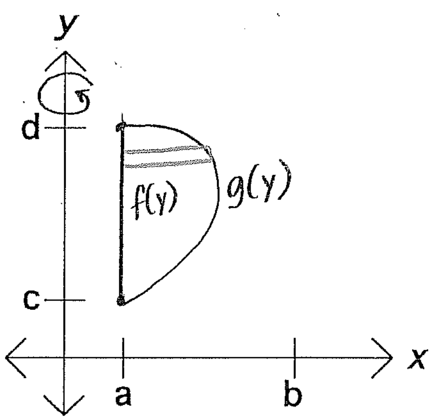
$$2\pi \int_{c}^{f} (d-y)(g(y)-f(y))\,dy$$

shell



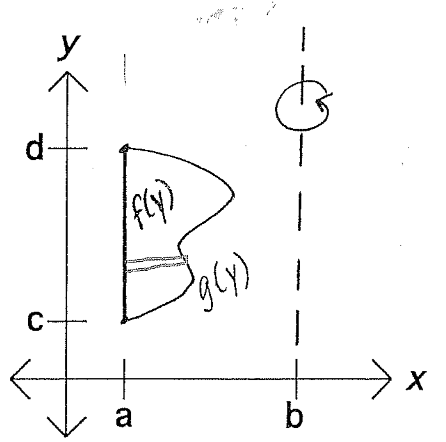
$$2\pi \int_{a}^{b} (e-x)(f(x)-g(x))\,dx$$

shell



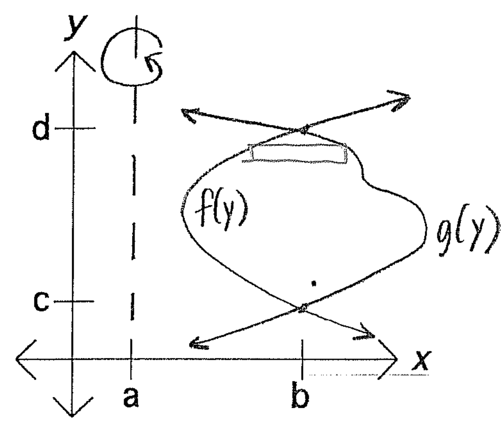
$$\pi \int_{c}^{d} [g(y)]^2-[f(y)]^2\,dy$$

washer



$$\pi \int_{c}^{d} \left((b-a)^2-[b-g(y)]^2\right)dy$$

washer

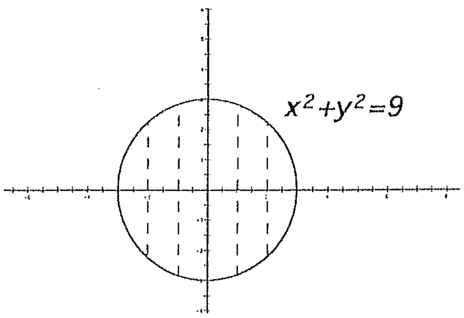


$$\pi \int_{c}^{d} \left([g(y)-a]^2-[f(y)-a]^2\right)dy$$

washer

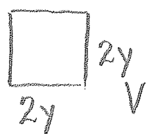
VOLUME OF KNOWN CROSS SECTIONS ON A BASE

Set up an integral to determine the volume of the solid generated by cross sections that are squares, equilateral triangles, and semi-circles on the given base.



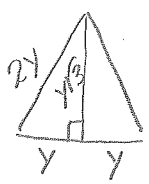
$y=\sqrt{9-x^2}$

$A=(2y)^2 = 4\sqrt{9-x^2}^2$



$V=2\int_0^3 4(\sqrt{9-x^2})^2\,dx = 8\int_0^3 9-x^2\,dx$

$= 8\left[9x-\frac{x^3}{3}\right]_0^3 = 8[27-9] = 8[18]$



$A=\frac{1}{2}(2y)(y\sqrt{3}) = y^2\sqrt{3} = \sqrt{3}\sqrt{9-x^2}^2$

$V=2\sqrt{3}\int_0^3 \sqrt{9-x^2}^2\,dx = 2\sqrt{3}\int_0^3 (9-x^2)\,dx = 2\sqrt{3}(18) = 36\sqrt{3}$

$A=\frac{1}{2}\pi y^2 = \frac{\pi}{2}\sqrt{9-x^2}^2$

$V=2\cdot\frac{\pi}{2}\int_0^3 \sqrt{9-x^2}^2\,dx = \pi(18) = 18\pi$