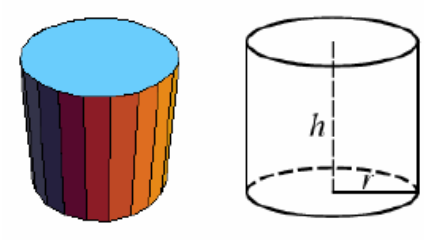
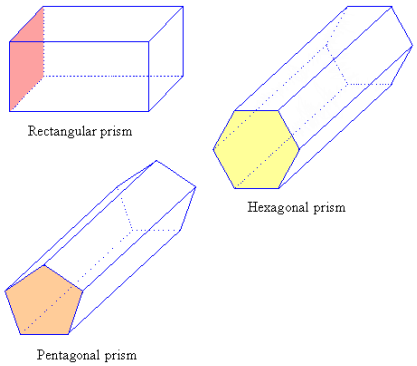
AP Calculus

5.5 – Volumes of Solids with Known Cross Sectional Areas

Volume of a Cylinder:



Volume of any prism with uniform width:



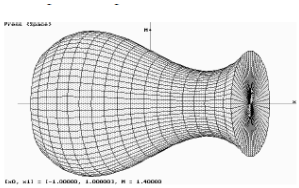
If a prism (or any solid) has uniform/same "width" all the way from one end to the other, then:

Total Volume = _____ X _____

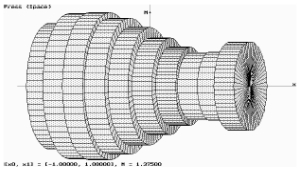
What if the solid does <u>not</u> have a uniform width all the way from one end to the other BUT has <u>circular cross sections</u>?

➔ **We have to use CALCULUS and volumes of revolution!!!!.....for example:**

To find volume of:



Slice it up into disks of width Δx_i and radius $f(x_i)$



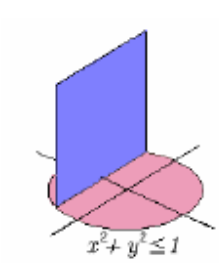
Volume of each disk = _____ → Approx. Total volume =

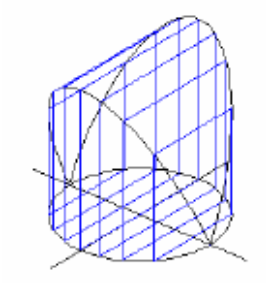
Exact Total Volume =

What if the solid does <u>not</u> have a uniform width all the way from one end to the other AND does <u>not</u> have circular cross sections either?

➔ **Can't use *"volumes of discs"* method ☹ ...but we can still use CALCULUS...yaaaaayyyy!!!**

Consider a solid whose <u>bottom</u> face is a circle and
every cross section perpendicular to the x-axis is a square.





How can we find the volume of this solid?

Remember: $\int_a^b stuff\ dx$ calculates the _____ of all the _____ between _____ and _____.

So, we just have to find _____.

→ Volume of solid = or

If the bottom face of the solid is the same shape as the area between 2 curves $f(x)$ and $g(x)$, then

Length of the BASE, *B*, of each cross section perpendicular to the x-axis =

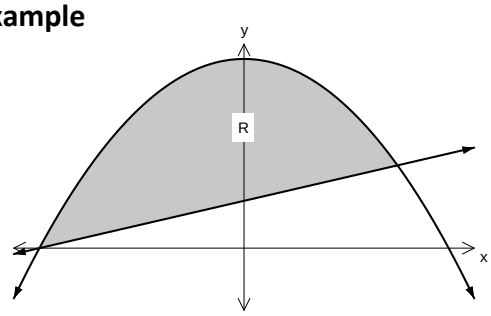
Now we can find the area of the cross-section of each slice in terms of B, for example:

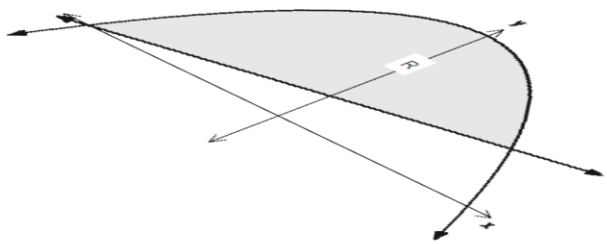
Area of a Square cross section:

Area of a Semi-circular cross section:

Area of a Right Isosceles Triangle cross section:

Example





The region *R* is the area bounded by the curves $f(x)=4-x^2$ and $g(x)=\frac{x}{2}+1$. *R* is the base of a solid and the cross sections of the solid are perpendicular to the x-axis. Determine the volume of the solid if the cross sections are all:

a. Squares

b. Right Isosceles Triangles with one isosceles side on *R*

c. Semi circles with the diameter on *R*

c. Rectangles with a height of 5 units