

Riemann Sums - Homework

For each problem, approximate the area under the given function using the specified number of rectangles/trapezoids. You are to do all 4 methods to approximate the areas.

#	Function	Interval	Number	Left Rectangles	Right Rectangles	Midpoint Rectangles	Trapezoids
1	$f(x) = x^2 - 3x + 4$	[1,4]	6	9.125	12.125	10.438	10.625
2	$f(x) = \sqrt{x}$	[2,6]	8	7.650	8.168	7.914	7.909
3	$f(x) = 2^x$	[0,1]	5	1.345	1.545	1.442	1.445
4	$f(x) = \sin x$	[0, π]	8	1.974	1.974	2.013	1.974

5. Roger decides to run a marathon. Roger's friend Jeff rides behind him on a bicycle and clocks his pace every 15 minutes. Roger starts out strong, but after an hour and a half he is so exhausted that he has to stop. The data Jeff collected is summarized below. Assuming that Roger's speed is always decreasing, estimate the distance that Roger ran in a) the first half hour and b) the entire race. (Trapezoids)

Time spent running (min)	0	15	30	45	60	75	90
Speed (mph)	12	11	10	10	8	7	0

a. $d \approx \frac{.25}{2}(12 + 22 + 10) = 5.5$ miles

b. $d \approx \frac{.25}{2}(12 + 22 + 20 + 20 + 16 + 14 + 0) = 13$ miles

6. Coal gas is produced at a gasworks. Pollutants in the air are removed by scrubbers, which become less and less efficient as time goes on. Measurements are made at the start of each month (although some months were neglected) showing the rate at which pollutants in the gas are as follows. Use trapezoids to estimate the total number of tons of coal removed over 9 months.

Time (months)	0	1	3	4	6	7	9
Rate pollutants are escaping (tons/month)	5	7	8	10	13	16	20

$$T \approx .5(5 + 7) + 1(7 + 8) + .5(8 + 10) + 1(10 + 13) + .5(13 + 16) + 1(16 + 20) = 103.5 \text{ tons}$$

7. For $0 \leq t \leq 1$, a bug is crawling at a velocity v , determined by the formula $v = \frac{1}{1+t}$, where t is in hours and v is in meters/hr. Find the distance that the bug crawls during this hour using 10 minute increments.

$$d \approx \frac{1}{2} \left(\frac{1}{6} \right) \left[v(0) + 2v\left(\frac{1}{6}\right) + 2v\left(\frac{2}{6}\right) + 2v\left(\frac{3}{6}\right) + 2v\left(\frac{4}{6}\right) + 2v\left(\frac{5}{6}\right) + v(1) \right] = 0.695 \text{ meters}$$

8. An object has zero initial velocity and a constant acceleration of 32 ft/sec^2 . Complete the chart to find the velocity at these specified times. Then determine the distance traveled in 4 seconds. $v = 32t$

t (sec)	0	.5	1	1.5	2	2.5	3	3.5	4
v (ft/sec)	0	16	32	48	64	80	96	112	128

$$d \approx \frac{1}{2} \left(\frac{1}{2} \right) \left[0 + 2(16) + 2(32) + 2(48) + 2(64) + 2(80) + 2(96) + 2(112) + 128 \right] = 256 \text{ feet}$$