FTC /19 **Name:** ____________________

Calculators permitted. Show All Work!!

1. The rate of water pouring into a tank is modeled by the function $E(t)=8$ gallons per hour and rate of water pouring out of the tank is modeled by the function $L(t)=10e^{-t}$ gallons per hour. If the tank contains 30 gallons of water at time $t=0$, determine each of the following.

a. Amount of water lost from the tank in the first 2.5 hours. [3 marks]

$$\int_0^{2.5} L(t)\,dt = 9.179 \text{ gallons.}$$

b. Amount of water remaining in the tank after 4 hours. [3 marks]

= 30 + Amt. of water poured in − Amt of water lost

$$= 30 + \int_0^4 8\,dt - \int_0^4 L(t)\,dt = 30 + 32 - 9.817$$

$$= 52.183 \text{ gallons}$$

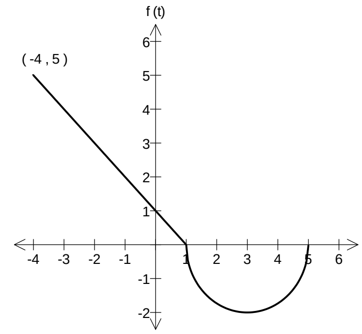
c. Setup, but do not evaluate, an integral equation that determines how long it takes for the tank to empty. [3 marks]

To empty the tank, the NET CHANGE in amt. of water must equal −30.

$$\Rightarrow \int_0^x 8\,dt - \int_0^x L(t)\,dt = -30$$

OR: $$\int_0^x 8 - L(t)\,dt = -30$$

2. The graph of $f(t)$ consists of a line segment and a semi-circle, as shown here:



If $g(x) = \int_1^x f(t)\, dt$ on $x \in [-4, 5]$, determine each of the following.

a. $g(1), g(3), g'(1), g'(3)$ [4 marks]

$g(1) = \int_1^1 f(t)\, dt = \underline{\underline{0}}$

$g(3) = \int_1^3 f(t)\, dt = -(\text{Area of } \frac{1}{4} \text{ of circle}) = -\frac{1}{4}\pi\, 2^2 = \underline{\underline{-\pi}}$

$g'(x) = f(x) \Rightarrow g'(1) = f(1) = \underline{\underline{0}}$

$g'(3) = f(3) = \underline{\underline{-2}}$

b. x-coordinate(s) of all local maxima of $g(x)$. Justify your answer. [2 marks]

local maxima occur when $g'(x) = 0$ and changes sign $+$ to $-$

$\Rightarrow f(x) = 0$ 〃 〃 〃 〃 〃 〃

$\Rightarrow \underline{\underline{x = 1}}$

c. Interval(s) on which $g(x)$ is concave up. Justify your answer. [2 marks]

$g(x)$ is cu when $g'(x)$ is increasing $\Rightarrow f(x)$ is incr.

$\Rightarrow x = \underline{\underline{(3, 5)}}$

d. x-coordinate(s) of all Points of Inflection of $g(x)$. Justify your answer. [2 marks]

$g(x)$ has PoI when $g'(x)$ changes from incr $\rightleftarrows$ decr

$\Rightarrow f(x)$ 〃 〃 〃 〃 〃

$\Rightarrow \underline{\underline{x = 3}}$