FTC Free Response GA
Solutions

2. (a) $\int_9^{17} E(t)\,dt = 6004.270$

6004 people entered the park by 5 pm.

$3\begin{cases} 1: & \text{limits} \\ 1: & \text{integrand} \\ 1: & \text{answer} \end{cases}$

(b) $15\int_9^{17} E(t)\,dt + 11\int_{17}^{23} E(t)\,dt = 104048.165$

The amount collected was \$104,048.

or

$\int_{17}^{23} E(t)\,dt = 1271.283$

1271 people entered the park between 5 pm and 11 pm, so the amount collected was

$\$15 \cdot (6004) + \$11 \cdot (1271) = \$104{,}041.$

1 : setup

(c) $H'(17) = E(17) - L(17) = -380.281$

There were 3725 people in the park at $t = 17$.

The number of people in the park was decreasing at the rate of approximately 380 people/hr at time $t = 17$.

$3\begin{cases} 1: & \text{value of } H'(17) \\ 2: & \text{meanings} \\ & 1: \text{ meaning of } H(17) \\ & 1: \text{ meaning of } H'(17) \\ & < -1 > \text{ if no reference to } t = 17 \end{cases}$

(d) $H'(t) = E(t) - L(t) = 0$

$t = 15.794$ or 15.795

$2\begin{cases} 1: & E(t) - L(t) = 0 \\ 1: & \text{answer} \end{cases}$

3. (a) $a(2) = v'(2) = -0.132$ or -0.133

1 : answer

(b) $v(2) = -0.436$

Speed is increasing since $a(2) < 0$ and $v(2) < 0$.

1 : answer with reason

(c) $v(t) = 0$ when $\tan^{-1}\left(e^t\right) = 1$

$t = \ln(\tan(1)) = 0.443$ is the only critical value for y.

$v(t) > 0$ for $0 < t < \ln(\tan(1))$

$v(t) < 0$ for $t > \ln(\tan(1))$

$y(t)$ has an absolute maximum at $t = 0.443$.

3 : { 1 : sets $v(t) = 0$; 1 : identifies $t = 0.443$ as a candidate; 1 : justifies absolute maximum }

(d) $y(2) = -1 + \int_0^2 v(t)\,dt = -1.360$ or -1.361

The particle is moving away from the origin since $v(2) < 0$ and $y(2) < 0$.

4 : { 1 : $\int_0^2 v(t)\,dt$; 1 : handles initial condition; 1 : value of $y(2)$; 1 : answer with reason }

4. (a) The function f is increasing on $[-3,-2]$ since $f' > 0$ for $-3 \le x < -2$.

2 : { 1 : interval; 1 : reason }

(b) $x = 0$ and $x = 2$

f' changes from decreasing to increasing at $x = 0$ and from increasing to decreasing at $x = 2$

2 : { 1 : $x = 0$ and $x = 2$ only; 1 : justification }

(c) $f'(0) = -2$

Tangent line is $y = -2x + 3$.

1 : equation

(d) $f(0) - f(-3) = \int_{-3}^{0} f'(t)\,dt$

$$= \frac{1}{2}(1)(1) - \frac{1}{2}(2)(2) = -\frac{3}{2}$$

$$f(-3) = f(0) + \frac{3}{2} = \frac{9}{2}$$

$$f(4) - f(0) = \int_0^4 f'(t)\,dt$$

$$= -\left(8 - \frac{1}{2}(2)^2\pi\right) = -8 + 2\pi$$

$$f(4) = f(0) - 8 + 2\pi = -5 + 2\pi$$

4 : {
1 : $\pm\left(\frac{1}{2} - 2\right)$ (difference of areas of triangles)

1 : answer for $f(-3)$ using FTC

1 : $\pm\left(8 - \frac{1}{2}(2)^2\pi\right)$ (area of rectangle − area of semicircle)

1 : answer for $f(4)$ using FTC }

5. (a) $g(-1) = \int_0^{-1} f(t)\,dt = -\int_{-1}^{0} f(t)\,dt = -\frac{3}{2}$

$g'(-1) = f(-1) = 0$

$g''(-1) = f'(-1) = 3$

3 { 1 : $g(-1)$
1 : $g'(-1)$
1 : $g''(-1)$

(b) g is increasing on $-1 < x < 1$ because $g'(x) = f(x) > 0$ on this interval.

2 { 1 : interval
1 : reason

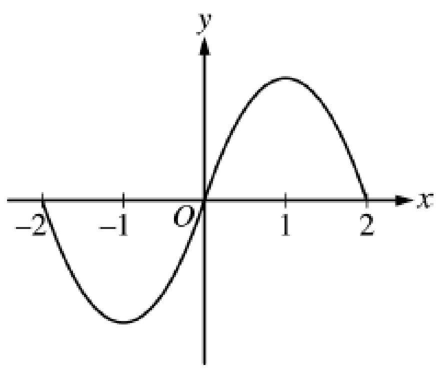
(c) The graph of g is concave down on $0 < x < 2$ because $g''(x) = f'(x) < 0$ on this interval.

or

because $g'(x) = f(x)$ is decreasing on this interval.

2 { 1 : interval
1 : reason

(d)



2 { 1 : $g(-2) = g(0) = g(2) = 0$
1 : appropriate increasing/decreasing and concavity behavior
$< -1 >$ vertical asymptote

6. (a) $\int_0^{12} H(t)\,dt = 70.570$ or 70.571

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $H(6) - R(6) = -2.924$,
so the level of heating oil is falling at $t = 6$.

1 : answer with reason

(c) $125 + \int_0^{12} (H(t) - R(t))\,dt = 122.025$ or 122.026

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(d) The absolute minimum occurs at a critical point or an endpoint.

$H(t) - R(t) = 0$ when $t = 4.790$ and $t = 11.318$.

The volume increases until $t = 4.790$, then decreases until $t = 11.318$, then increases, so the absolute minimum will be at $t = 0$ or at $t = 11.318$.

$$125 + \int_0^{11.318} (H(t) - R(t))\,dt = 120.738$$

Since the volume is 125 at $t = 0$, the volume is least at $t = 11.318$.

$3 : \begin{cases} 1 : \text{sets } H(t) - R(t) = 0 \\ 1 : \text{volume is least at } t = 11.318 \\ 1 : \text{analysis for absolute minimum} \end{cases}$