

AP Calculus AB
4.5 - Group Assignment - FR Questions on FTC

Questions marked * are Non-Calculator.

*1. The table below shows the speedometer readings of a car at several time intervals over $0 \leq t \leq 5$ hours.

t (hr)	0	1	2	3	4	5
$v(t)$ (miles/hr)	10	40	50	30	80	70

Estimate the value of $\int_0^5 v(t) dt$ using:

- Five trapezoids
- MRAM, $N=5$

Use correct units for the answers and interpret your answers.

2. The rate at which people enter a park on a given day is modeled by the function, $E(t) = \frac{15600}{t^2 - 24t + 160}$. The rate at which people leave the same park on the same day is modeled by the function, $L(t) = \frac{9890}{t^2 - 38t + 370}$. Both $E(t)$ and $L(t)$ are measured in people per hour, and time t is measured in hours after midnight. These functions are valid for $9 \leq t \leq 23$, the hours during which the park is open. At time, $t = 9$, there are no people in the park.

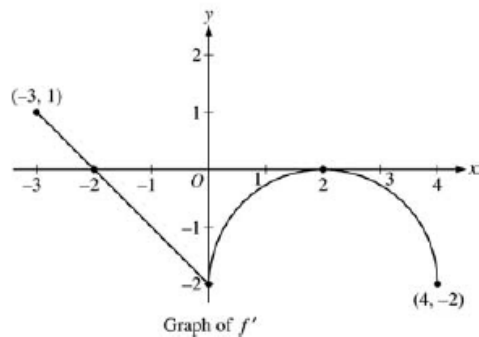
- How many people have entered the park by 5:00pm ($t=17$)? Round your answer to the nearest whole number.
- The price of admission to the park is \$15 until 5:00pm ($t=17$). After 5:00pm the price of admission is \$11. How much money is collected from admissions to the park on the given day?
- Let $H(t) = \int_9^t E(x) - L(x) dx$ for $9 \leq t \leq 23$. Explain the meaning of $H(17) = 3725$.
- Determine the value of $H'(17)$ and interpret the meaning of your answer.
- At what time(s) over $9 \leq t \leq 23$ are there maximum people in the park?

3. A particle moves along the y -axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$.

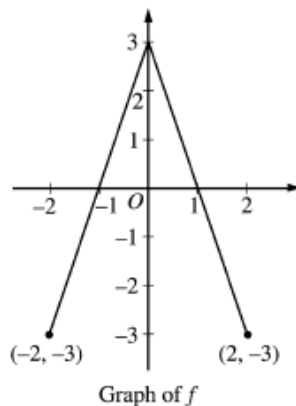
At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

- Find the acceleration of the particle at time $t = 2$.
- Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.
- Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.
- Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

- *4. Let f be a function defined on the closed interval $-3 \leq x \leq 4$ with $f(0) = 3$. The graph of f' , the derivative of f , consists of one line segment and a semicircle, as shown above.



- (a) On what intervals, if any, is f increasing? Justify your answer.
- (b) Find the x -coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.
- (c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.
- (d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.
- *5. The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t) dt$.
- (a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.
- (b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.
- (c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.
- (d) On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$.



6. A tank contains 125 gallons of heating oil at time $t = 0$. During the time interval $0 \leq t \leq 12$ hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t+1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12 \sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

- (a) How many gallons of heating oil are pumped into the tank during the time interval $0 \leq t \leq 12$ hours?
- (b) Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Give a reason for your answer.
- (c) How many gallons of heating oil are in the tank at time $t = 12$ hours?
- (d) At what time t , for $0 \leq t \leq 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.