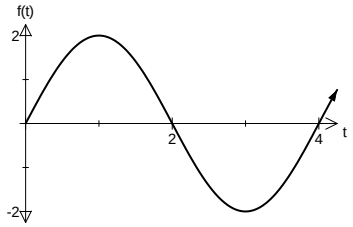
AP Calculus

4.4A – Investigating the Relationship Between

"Area under $f(t)$" and $f(t)$ itself!

The graph of $f(t)=2\sin\left(\frac{\pi t}{2}\right)$, $0\le t\le 4$ is given on the right:



1. Let $F(x)=\int_0^x f(t)\,dt$

Complete the following table and sketch the graph of $y=F(x)$.

x	0	0.5	1	1.5	2	2.5	3	3.5	4
$F(x)$									

2. Let $G(x)=\int_2^x f(t)\,dt$

Complete the following table and sketch the graph of $y=G(x)$.

x	0	0.5	1	1.5	2	2.5	3	3.5	4
$F(x)$									

3. Let $H(x)=\int_4^x f(t)\,dt$

Complete the following table and sketch the graph of $y=H(x)$.

x	0	0.5	1	1.5	2	2.5	3	3.5	4
$F(x)$									

4.

	$F(x)$	$G(x)$	$H(x)$
x-coordinate of max. value of AREA function			
x-coordinate of min. values of AREA function			
Intervals on which AREA function is increasing			
Intervals on which AREA function is decreasing			

5. Compare and contrast the graphs of $y = F(x)$, $y = G(x)$, $y = H(x)$.

6. Considering only the AREA function, $y = F(x)$. Explain in terms of the **accumulation of area** why $y = F(x)$,

a. Should have a maximum at the x-coordinate you stated in Q.4.

b. Should have a minimum at the x-coordinate you stated in Q.4.

c. Should be increasing on the interval you stated in Q.4.

d. Should be decreasing on the interval you stated in Q.4.

7. When $F(x), G(x), H(x)$ have a maximum, $f(t)$ changes _____.

When $F(x), G(x), H(x)$ have a minimum, $f(t)$ changes _____.

When $F(x), G(x), H(x)$ are increasing, $f(t)$ is _____.

When $F(x), G(x), H(x)$ are decreasing, $f(t)$ is _____.

When $F(x), G(x), H(x)$ have a POI, $f(t)$ has _____.

8. Based on your responses to Q.7, and everything you know about the graphs functions and their derivatives, what can you conclude about the relationship between the function that represents the AREA under the graph of $f(t)$ (i.e. $F(x)$ or $\int_a^x f(t)\,dt$) and the function $f(t)$ itself. Explain fully.

9. Complete the following statements:

The _____ of the AREA function gives $f(t)$ → _____ = _____

→ The AREA accumulated under $f(t)$ from $t=0$ to $t=x$ is the _____ of $f(t)$ at $t=x$

→ _____ = _____

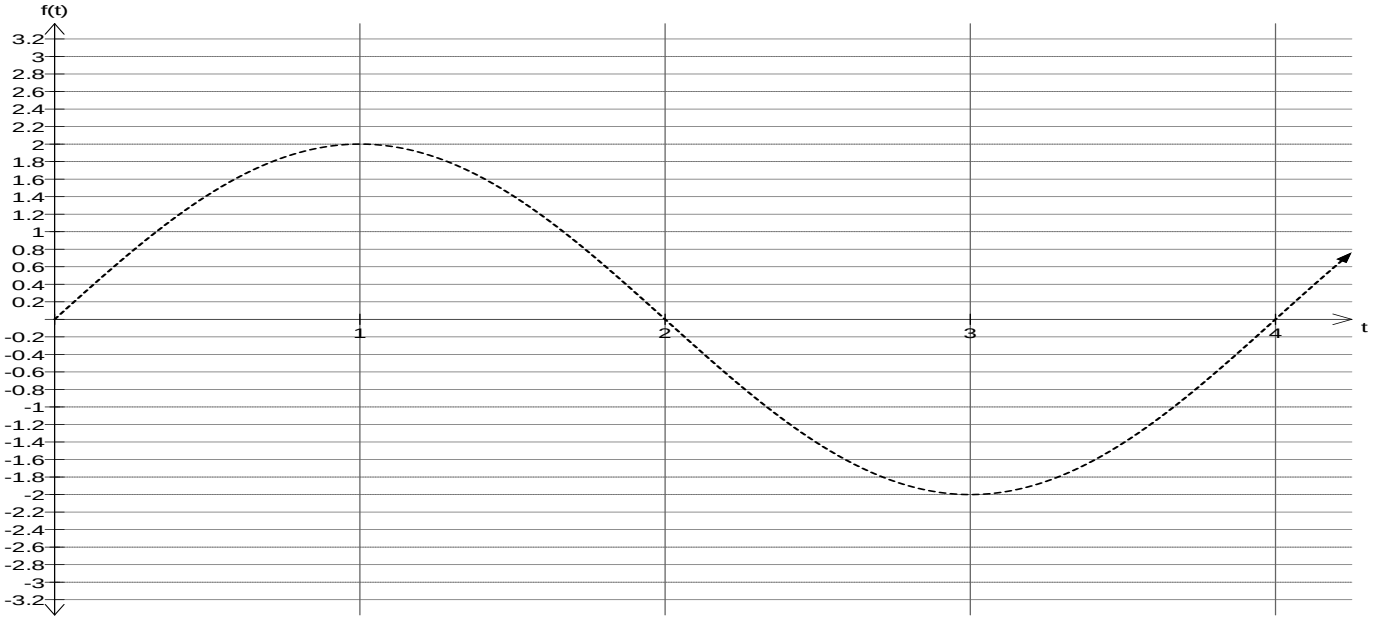
10. We have shown that the graphs of $y=F(x),\ y=G(x),\ y=H(x)$ differ only by a vertical shift of some constant value. How do they differ exactly?

If the graph of $\int_0^x f(t)\,dt = F(x)$ is used as the reference graph and $F(x)$ = "area from 0 to x", then:

$$G(x)=\int_2^x f(t)\,dt = F(x) - \_\_\_\_\_$$

$$H(x)=\int_4^x f(t)\,dt = F(x) - \_\_\_\_\_$$

→ In general, $\int_a^x f(t)\,dt = F(x) - \_\_\_\_\_$ or $\int_a^b f(t)\,dt = F(\ \ ) - F(\ \ )$


f(t)
3.2
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
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-2
-2.2
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-2.8
-3
-3.2
1
2
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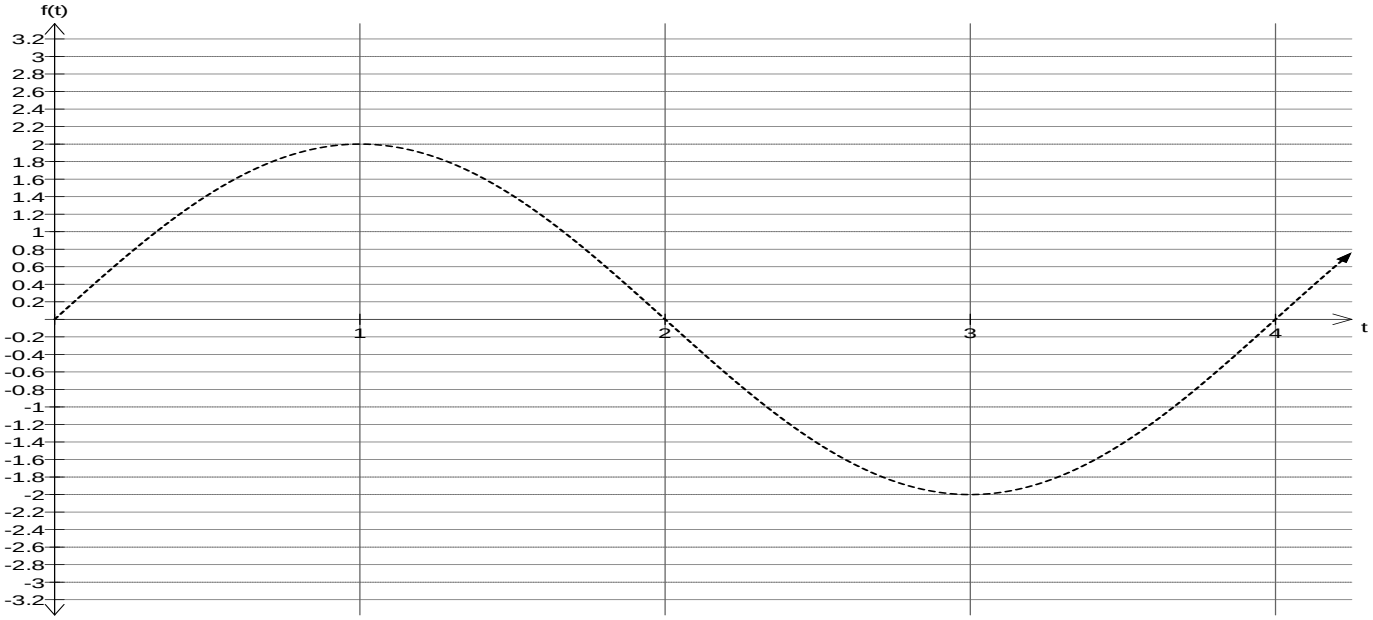
4
t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2.8
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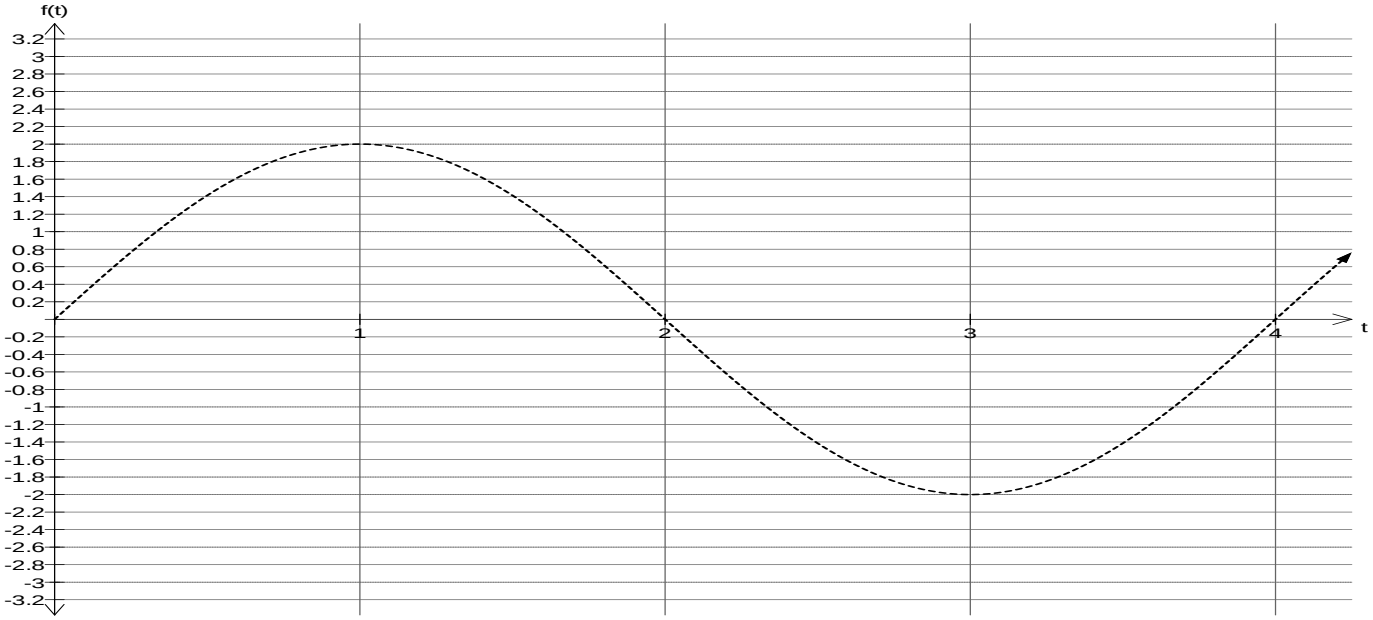
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
-2
-2.2
-2.4
-2.6
-2.8
-3
-3.2
1
2
3
4
t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3.2
3
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-3
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4
t