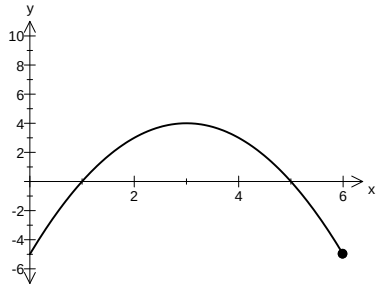
AP Calculus

4.4A – Investigating the Relationship Between "Area under $f(t)$" and $f(t)$ itself!

Homework Assignment



The graph of $f(t) = -(x-3)^2 + 4$, $0 \le t \le 6$ is given on the right:

1. Describe how you think the graph of $F(x) = \int_0^x f(t)\,dx$ will behave in the following intervals, and why:

   a. $0 \le t \le 1$

   b. $1 \le t \le 3$

   c. $3 \le t \le 5$

   d. $5 \le t \le 6$

2. Predict when the graph of $y = F(x)$ will have a min, max, and POI. Justify your prediction.

3. Complete the following table and sketch the graph of $y = F(x)$. [*You will need your own graph paper…* ☹]

x	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
$F(x)$													

4. Compare and contrast the graphs of $y = F(x)$ and $y = f(t)$ in terms of max/min, increasing/decreasing, POI, etc.

5. Describe how you think the graph of $G(x)=\int_2^x f(t)\,dx$ will behave in the following intervals, and why:

a. $0 \le t \le 1$

b. $1 \le t \le 3$

c. $3 \le t \le 5$

d. $5 \le t \le 6$

5. Predict when the graph of $y=G(x)$ will have a min, max, and POI. Justify your predictions

6. Complete the following table and sketch the graph of $y=G(x)$.

x	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
$G(x)$													

7. Compare and contrast the graphs of $y=G(x)$ and $y=f(t)$ in terms of max/min, increasing/decreasing, POI, etc.

8. Compare and contrast the graphs of $y=F(x)$ and $y=G(x)$. Give reasons for their similarities and differences.

Evaluating Definite Integrals Using Antiderivatives.

In class, we proved the following : *If* $F(x)$ *is the antiderivative of* $f(x)$*, then:*

(Area accumulated under $y = f(t)$ *from* $t = 0$ *to* $t = x$*) = (Value of the antiderivative of* $f(t)$*, at* $t = x$*) =* $F(x)$

$$\rightarrow \int_0^x f(t)\ dt = F(x) \quad \textit{where,}\ F(x) = \int f(x)\,dx \quad \textit{or}\ F'(x) = f(x)$$

We can use this FUNDAMENTAL THEOREM OF CALCULUS to evaluate, $\int_a^b f(t)\ dt$. How??...Watch and learn…☺

$$\int_a^b f(t)\ dt = \int_0^b f(t)\ dt - \int_0^a f(t)\ dt = F(b) - F(a)$$

Example: Determine the area under the graph of $f(t) = t^2$ **from** $t = 1$ **to** $t = 4$**. [Means:** Evaluate: $\int_1^4 t^2\ dt$ **]**

$\int_1^4 t^2\ dt$ = (*Antiderivative of* t^2 *at* $t = 4$*) –* (*Antiderivative of* t^2 *at* $t = 1$*)* [*Antiderivative of* $t^2 = \frac{t^3}{3}$*]*

$$\rightarrow \int_1^4 t^2\ dt = \left.\frac{t^3}{3}\right]_{t=1}^{t=4} = \frac{4^3}{3} - \frac{1^3}{3} = 21$$

Check using: $fnInt(X^2, X, 1, 4) = 21$ *YAAAAYYYY!!!*

Evaluate the following definite integrals using antiderivatives and then verify your answer using $fnInt(\ )$**.**

1. $\int_2^6 t^2\ dt$

2. $\int_1^4 t\ dt$

3. $\int_{\pi}^{\frac{3\pi}{2}} \mathrm{Sin}(t)\ dt$

4. $\int_2^5 3u^4 + 2u^2 - 2u\ du$