

PM-Calculus

4.4 - Applications of the Derivative and the Integral

Average ROC of $f(x)$ over $x = [a, b]$:

Instantaneous ROC of $f(x)$ at $x = c$:

Linear Motion is: _____

$x(t)$ or $s(t)$:

$x'(t)$ or $s'(t)$:

$x''(t)$ or $s''(t)$:

Average ROC of position of an object , $x(t)$, on $t = [t_1, t_2]$:

Average Velocity of object on $t = [t_1, t_2]$:

Given, $f'(x)$, the **Net Change in the Value of** $f(x)$ on the interval, $x = [a, b]$ is:

→ **Net Change in Position** on $t = [t_1, t_2]$:

Final position of an object at any time $t = c$:

Total Distance travelled by an object on $t = [t_1, t_2]$:

Examples

1. A particle is moving on a line such that its position at time t is given by, $x(t) = 3t^2 + t$ miles.
 - a. Find the average velocity of the particle over $t = [1, 3]$.
 - b. Find the Instantaneous velocity at $t = 1$.
 - c. Find the change in position over $[4, 9]$.
 - d. Find the acceleration at $t = 5$.

2. A particle is moving on a line such that its position at time t is given by, $x(t) = 3t^3 - 4t^2 + 6t - 1$ km.
 - a. Find the average velocity of the particle over $t = [4, 7]$.
 - b. Find the Instantaneous velocity at $t = 4$.
 - c. Find the change in position over $[3, 9]$.
 - d. Find the acceleration at $t = 10$.

3. A particle is moving on a line such that its position at time t is given by, $x(t) = \sqrt{1 + \ln^2(t)}$ miles.
 - a. Find the average velocity of the particle over $t = [1, 3]$.
 - b. Find the Instantaneous velocity at $t = 1$.
 - c. Find the change in position over $[4, 9]$.
 - d. Find the acceleration at $t = 5$.

4. The amount of a certain drug in the blood stream is given by, $c(t) = e^{\frac{1}{t}}$, where $c(t)$ is the concentration of the drug in $\mu\text{g/L}$ and t is time in hours.
 - a. Interpret the meaning of $c'(t)$.
 - b. Find the concentration of the drug after 4 hours.
 - c. Find the rate at which the drug concentration is changing after 4 hours. Is the concentration increasing or decreasing at this time? Explain.
 - d. Find the average ROC of drug concentration over $t = [3, 7]$ hours.

5. An object is moving in a straight line with velocity, $v(t) = 4t^2 - 3t + 1$ m/s.
- Find the velocity of the object after 5 seconds.
 - Find the acceleration after 5 seconds.
 - Find the displacement of the object over $t = [2, 5]$.
 - If the object is at 3m at time $t = 0$, find its position at time $t = 4s$.
 - Find the total distance travelled by the object in the first 4 seconds.
6. A population of insects increases at a rate of $R(t) = 200 + 10t + 0.25t^2$ insects per day. Find the insect population after 3 days, assuming that there are 35 insects at time $t = 0$.
7. The rate at which a drug is being absorbed by the body is given by, $r(t) = e^{\frac{1}{t}}$ $\mu\text{g} / \text{L}$. How many μg of the drug is absorbed by the body in the first 3 hours?