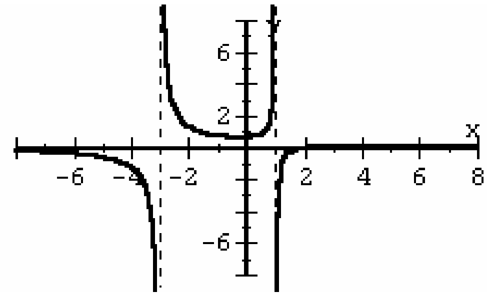
Horizontal and Slant Asymptotes

Case 1

If deg(Denominator) > deg(Numerator), then the Horizontal Asymptote is $y = 0$.

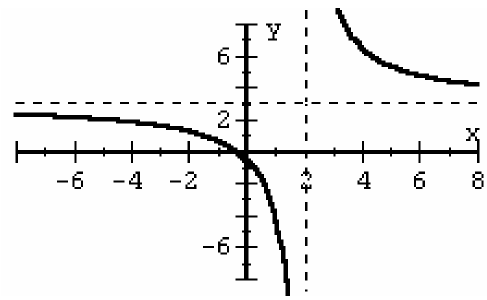
ex) $f(x) = \dfrac{x-2}{x^2+2x-3} = \dfrac{x-2}{(x-1)(x+3)}$



Case 2

If deg(Numerator) = deg(Denominator), then the Horizontal Asymptote is $y = \dfrac{a_n}{b_n}$ where $y = \dfrac{a_n}{b_n}$ is the ratio of the leading coefficients.

ex) $f(x) = \dfrac{3x+1}{x-2} = \dfrac{\boxed{3}\,x+1}{\boxed{1}\,x-2}$



Horizontal asymptote is $y = \dfrac{3}{1} = 3$

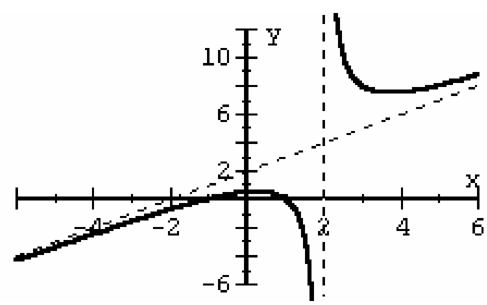
Case 3

If deg(Numerator) = deg(Denominator) + 1, then the graph has a Slant Asymptote.
An equation of the Slant Asymptote is $y = mx + b$, where m and b may be determined by long division.

ex) $f(x) = \dfrac{x^2-1}{x-2} = \boldsymbol{x+2} + \dfrac{3}{x-2}$

$$\begin{array}{r} x + 2 \\ x - 2\,\big)\overline{x^2 + 0x - 1} \\ -\,(x^2 - 2x) \\ \hline 2x - 1 \\ -\,(2x - 4) \\ \hline 3 \end{array}$$

$x + 2$ ← slant asymptote



Equation of the Slant asymptote is: $y = x + 2$

Fact: The graph of a rational function will NEVER cross its vertical asymptote, but May cross its horizontal or slant asymptote. (see Case 1, the graph crosses its horizontal asymptote at $x = 2$)