PM - Calculus
3.4 - Related Rates

What is a **Related Rates** problem?

Example 1

One face of a plant cell has a rectangular shape with an initial length of $3\mu m$ and width of $2\mu m$. As the cell absorbs water, the length of the face increases at a rate of $1\mu m$ per second and the width increases at a rate of $2\mu m$ per second. Determine how fast the area of the face of the cell is changing after 2 seconds.

Algebraic Method

Diagram   Given   Find

Relationship/Equation/Formula   Differentiate with respect to t

Substitute values to determine answer

Example 2

The volume of a sphere is given by, $V = \dfrac{4}{3}\pi r^3$ and its surface area is given by, $S = 4\pi r^2$. The radius of the sphere is expanding at a rate of 14 inches/min.

a. Determine the rate of change of change of the volume wrt to time when the radius is 8 inches.

b. Determine the rate of change of the surface area wrt to time when the radius is 8 inches.

c. Determine the rate of change of the volume wrt to time at $t = 2$ min, assuming that the radius is 3 inches at time, $t = 0$.

Example 3

A 10 foot ladder is leaning against a brick wall. The top of the ladder is originally 8.5 feet above the ground. The top of the ladder begins to fall at a <u>constant</u> rate of 0.5 feet per second. At what rate is the <u>foot</u> of the ladder moving when the top of the ladder is 5.5 feet from the ground?

Example 3

A spy is tracking a rocket launch 10km away, through a telescope to determine its velocity. At a certain moment, the angle between the ground and the spy's telescope is $\frac{\pi}{3}$ radians and is increasing at a rate of 0.5 radians/min. What is the rocket's velocity at that moment?

Example 4

Train A is travelling west towards Vancouver at 120mph, while train B is travelling north, away from Vancouver, at 90mph. At time, $t=0$, train A is 10 miles east of Vancouver, and train B is 20 miles north of Vancouver. Calculate the rate at which the distance between the trains is changing at time, $t=10\min$.