

AP Calculus
Unit 3
AP Free Response Review Questions

Justify every answer!! Justify every answer!! Justify every answer!! Justify every answer!! Justify every answer!!

1. $f(x) = \sin^2 x - \sin x$; $0 \leq x \leq \frac{3\pi}{2}$.

- Find the x -intercepts of f . (NC)
- Find the intervals on which f is increasing. (GC)
- Find x -coordinates of all local maxima and local minima. (GC)
- Find the absolute maximum and minimum values of f . (GC)

2. $f(x) = x + \sin^2 x$; $\frac{\pi}{6} \leq x \leq \frac{5\pi}{6}$. (GC)

- Find exact values of x for which $f'(x) = 1$.
- Find exact values for the x -coordinates of all minimum points of f .
- Find exact values for the x -coordinates of all POI.

3. (NC) $f(x)$ is continuous on the interval $[0, 4]$. The properties of f, f', f'' are indicated in the table below.

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

- Find the x -coordinates of all local maxima and minima.
- Find the x -coordinates of all POI.
- Determine the equation of the line tangent to f at $x = 0$.
- Sketch a possible graph of f .

4. (NC) $f(x)$ is continuous on the interval $[-3, 3]$ and $f(-3) = 1$ and $f(3) = 4$. The properties of f', f'' are indicated in the table below.

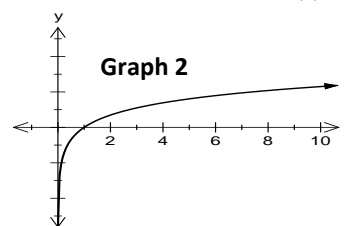
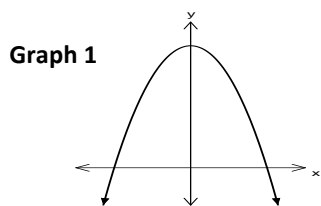
x	$-3 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$
$f'(x)$	Positive	Does not exist	Negative	0	Negative
$f''(x)$	Positive	Does not exist	Positive	0	Negative

- What are the x -coordinates of any local extrema?
- What are the x -coordinates of any POI?
- Sketch a possible graph of $f(x)$.
- Determine the x -coordinates of the absolute maximum and minimum.

5. **(NC)** $f(x)$ is continuous on the interval $[-3,3]$. The properties of f, f', f'' are indicated in the table below.

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3
$f(x)$	1	+ve	0	-ve	-1	-ve	0
$f'(x)$	0	-ve	-2	-ve	u/d	+ve	1
$f''(x)$	0	-ve	0	+ve	u/d	-ve	4

- Find the x -coordinates of all local maxima and minima.
 - Find the x -coordinates of all POI.
 - Determine the x -coordinates of any cusp/corner.
 - Determine the equation of the line tangent to f at $x=1$.
 - Sketch a possible graph of f on $x \in [-3,3]$ given that f is an **even** function.
6. a. Consider **Graph 1** of f given below. Explain why $f''(0) < f'(0) < f(0)$.
- b. Consider **Graph 2** of f given below. Arrange the following values in increasing order: $f(1), f'(1), f''(1)$.



7. Sketch the graph of f that has the following properties:
 $f(0) = f(2) = 0$, $f'(x) > 0$ for $x < 1$, $f'(1) = 0$, $f'(x) < 0$ for $x > 1$, $f''(x) < 0$
8. **(NC)** A cubic polynomial function is defined as $f(x) = x^3 + ax^2 + bx + c$ where a, b , and c are constants. The function f has a local minimum at $x=5$, a POI at $x=2$, and $f(-3)=5$. Determine the values of the constants.
9. **(GC)** A function is defined as $f(x) = ax^3 + bx^2 + cx + d$ and has a relative maximum at $(2, 4)$, a relative minimum at $(4, 2)$, and a POI at $(3, 3)$. Determine the values of a, b, c , and d .
10. **(NC)** $h(x)$ is a function that is defined for all $x \neq 0$ and $h(4) = -3$. The derivative of $h(x)$ is given by $h'(x) = \frac{x^2 - 2}{x}$ for all $x \neq 0$.
- Find all values of x for which the graph of $h(x)$ has a vertical asymptote, a horizontal tangent, and determine if $h(x)$ has a local maximum, local minimum, or neither at each of these values.
 - On what intervals, if any, is the graph of $h(x)$ concave up?
 - Find an equation for the tangent to $h(x)$ at $x=4$. Does the tangent line lie above or below the graph?
11. **(NC)** Let $f(x) = k\sqrt{x} - \ln x$ for $x > 0$, where k is a positive constant.
- Find $f'(x), f''(x)$.
 - For what value(s) of the constant k does f have a critical point at $x=1$? For this value of k , determine whether f has a relative minimum, relative maximum, or neither at $x=1$.
 - For another value of the constant k , the graph of f has a POI on the x-axis. Find this value of k .

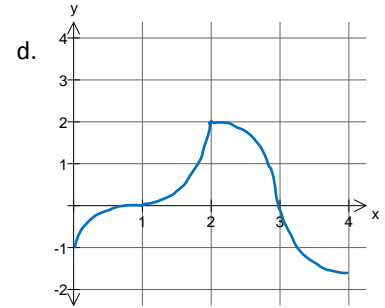
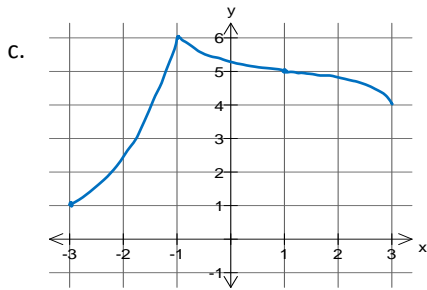
Solutions

1. a. $x - \text{int} = 0, \frac{\pi}{2}, \pi$ b. Increasing on $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$ and $\left(\frac{5\pi}{6}, \frac{3\pi}{2}\right)$
 c. Local min: $x = \frac{\pi}{6}, \frac{5\pi}{6}$, Local max: $x = \frac{\pi}{2}$ d. Abs. max: $f\left(\frac{3\pi}{2}\right) = 2$, Abs. min: $f\left(\frac{\pi}{6}\right), f\left(\frac{5\pi}{6}\right) = -\frac{1}{4}$

2. a. $x = \frac{\pi}{2}$ b. Critical Value at $x = \frac{3\pi}{4}$, but no local max/min. ($f' \geq 0, \forall x$) c. POI: $x = \frac{\pi}{4}, \frac{3\pi}{4}$

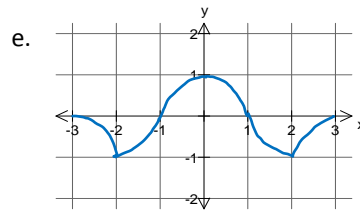
3. a. Local max: $x = 2$ b. POI: $x = 1, 2, 3$ c. $y = 4x - 1$

4. a. Local max: $x = -1$ b. POI: $x = 1$ d. Abs. max: $x = -1$
 Abs. min: $x = -3$



5. a. Local min: $x = 2$ b. POI: $x = 1, 2$ c. Cusp at $x = 2, f'(2) = \text{u/d}$ but $f(2) = \text{defined}$.

d. $y = -2x + 2$



6. a. At $x = 0: f$ is CD $\rightarrow f''(0) < 0, f'(0) = 0, f(0) > 0$ b. $f''(1) < f(1) < f'(1)$

7. 8. $a = -6, b = -15, c = 41$ 9. $a = 0.5, b = -4.5, c = 12, d = -6$

10. a. Vertical Asymptote: $x = 0$, Horizontal tangent: $x = \pm\sqrt{2}$, Local min: $x = \pm\sqrt{2}$

- b. $f''(x) > 0, \forall x \rightarrow f(x)$ is CU for all x c. $y = \frac{14}{4}(x-4) - 3$, tangent is below f

11. a. $f'(x) = \frac{k}{2\sqrt{x}} - \frac{1}{x}, f''(x) = -\frac{1}{4}kx^{-\frac{3}{2}} + x^{-2}$ b. $k = 2, f'(1) = \text{local min}$ (2nd derivative test) c. $k = \frac{4}{e^2}$