AP Calculus
Unit 2 - Review II

1. Given, $f(x)=\tan(\cos x)$. Find $f'(x)$.

2. Given, $y=\sin\left(\tan\sqrt{\sin x}\right)$. Find $\frac{dy}{dx}$.

3. Given, $s=2\theta\sqrt{\sec\theta}$. Find $\frac{ds}{d\theta}$.

4.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	5	5	e	$\sqrt{2}$
5	2	8	π	7

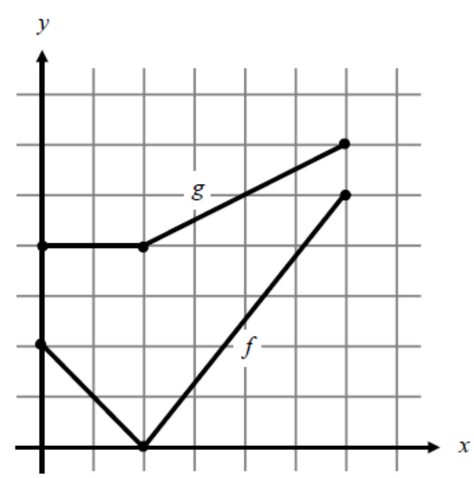
Use the table above to answer the following:

a. If, $h(x)=f[g(x)]$ determine $h'(2)$.

b. If, $h(x)=g[f(x)]$ determine $h'(2)$.

c. If, $h(x)=f[f(x)]$ determine $h'(2)$.

5. Use the graphs of $f(x)$ and $g(x)$ to answer the following. [Important question - make sure you "get it" all !!]



a. If, $r(x)=f[g(x)]$, determine $r'(1)$.

b. If, $s(x)=g[f(x)]$, determine $s'(4)$.

c. Determine, $\lim\limits_{h\to 0}\frac{g(4+h)-g(4)}{h}$ and interpret its meaning.

d. Determine, $\lim\limits_{x\to 1}\frac{f(1)-f(x)}{1-x}$ and interpret its meaning.

e. Determine, $\frac{g(4)-g(1)}{4-1}$ and interpret its meaning.

f. Determine, $\lim\limits_{h\to 0}\frac{f(2+h)-f(2)}{h}$ and interpret its meaning.

g. Determine, $\lim\limits_{h\to 0^+}\frac{g(6+h)-g(6)}{h}$ and explain your answer.

h. Determine, $\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$.

6. If, $y=\tan(x)$, determine $\frac{dy}{dx}$.

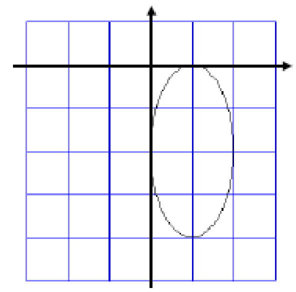
7. Find the equation of the tangent to $f(x)=\sqrt{25-x^2}$ at $x=4$.

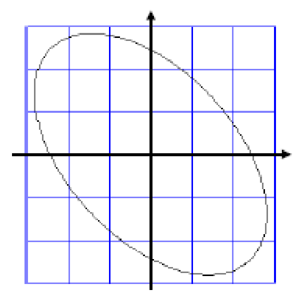
8. Given, $y=\sin^4(3x)$, determine $\frac{dy}{dx}$.

9. Determine, <u>algebraically</u>, the coordinates of the points on the graph of

$4x^2+y^2-8x+4y+4=0$ where there is a vertical tangent.

Does your answer make sense, given the graph on the right?





10. Determine, <u>algebraically</u>, the coordinates of the points on the graph of

$x^2+xy+y^2=6$ where there is a horizontal tangent.

Does your answer make sense, given the graph on the right?

11. Determine the equation of the tangent line to the graph of $y=\tan x$ at $x=\frac{\pi}{4}$.

12. Determine the equation of the tangent line to the graph of $y=\tan^{-1}x$ at $x=1$.

13. Given, $f(x)=x^3+2x-1$, determine the value of $\left(f^{-1}\right)'(2)$.

14. Given, $f(x)=x^3-\frac{2}{3}$ and $f\left[g(x)\right]=x$, determine the value of $g'(6)$.

15. Given, $g(t)=\ln(\ln t)$, determine $g'(t)$.

16. Given, $y=\ln\left(\frac{1+x^2}{1-x^3}\right)$, determine $\frac{dy}{dx}$.

17. Given, $y=x^{\ln x}$, use Log Differentiation to determine $\frac{dy}{dx}$.

18. Given, $f(x)=\left(x^2+1\right)^{(2-3x)}$, use Log Differentiation to determine $f'(1)$ **without the use of a calculator**.

19. If, $f(x)=\tan x$ and $g(x)=x^2$ determine all $x\in[0,\pi]$ for which $f(x)$ and $g(x)$ have tangents that are parallel. [*GC permitted*]

20. Determine the value of: $\lim_{h\to 0}\frac{2(x+h)^5-5(x+h)^3-2x^5+5x^3}{h}$ [*Non-calculator*]

21. Given, $f(x)=\begin{cases}\sin x & ;\ x<2\\ 3x^2+n & ;\ x\geq 2\end{cases}$

a. Determine the value of *n* that makes $f(x)$ continuous at $x=2$.

b. Does the value of *n* that made $f(x)$ continuous at $x=2$, make $f(x)$ differentiable at $x=2$? Justify your answer.

c. What do the answers to parts (a) and (b) tell us about *continuity* and *differentiability*?

Solutions

1. $f'(x) = -\sec^2(\cos x)\sin x$

2. $\frac{dy}{dx} = \cos\left(\tan\sqrt{\sin x}\right)\sec^2\left(\sqrt{\sin x}\right)\frac{1}{2\sqrt{\sin x}}\cos(x)$

3. $\frac{ds}{d\theta} = 2\sqrt{\sec\theta} + 2\theta\frac{1}{2\sqrt{\sec\theta}}\sec\theta\tan\theta$

4. $a)\ \pi\sqrt{2} \quad b)\ 7e \quad c)\ \pi e$

5. $a)\ r'(1) = 0 \quad b)\ s'(4) = \frac{5}{8} \quad c)\ \frac{1}{2}; g'(4)$

   $d)\ -1; f'(1) \quad e)\ \frac{1}{3}; av.roc[1,4] \quad f)\ dne;\ f'(2)$

   $g)\ dne;\ not\ RH-diff.@\ x=6 \quad h)\ f'(x) = \begin{cases} -1 & ; x = (0,2) \\ \frac{5}{4} & ; x = (2,6) \end{cases}$

6. $\frac{dy}{dt} = \sec^2(x)\frac{dx}{dt}$

7. $y_T = -\frac{4}{3}(x-4)+3$

8. $\frac{dy}{dx} = 12\sin^3(3x)\cos(3x)$

9. $(0,-2)$ *and* $(2,-2)$

10. $(1.41, -2.82)$ *and* $(-1.41, 2.82)$

11. $y_T = 2\left(x - \frac{\pi}{4}\right)+1$

12. $y_T = \frac{1}{2}(x-1) + \frac{\pi}{4}$

13. $\left(f^{-1}\right)'(2) = \frac{1}{5}$

14. $g'(6) = 0.094$

15. $g'(t) = \frac{1}{t\ln t}$

16. $\frac{dy}{dx} = \frac{2x}{1+x^2} + \frac{3x^2}{1-x^3}$

17. $y' = x^{\ln x}\frac{2\ln x}{x}$

18. $y'(1) = -\frac{1+3\ln 2}{2}$

19. $x = 2.083$

20. $10x^4 - 15x^2$

21. $a)\ n = -11.97 \quad b)$ *No, because* $f'(2^-) \neq f'(2^+)$

    $c)$ *Continuity at* $x = c$ *does not guarantee differentiability at* $x = c$