

AP Calculus

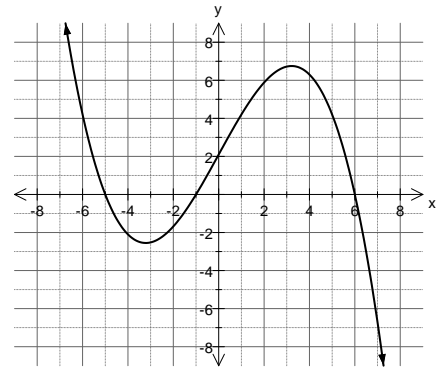
2.12A - Related Rates I

Explain the meaning of: $\left. \frac{dy}{dx} \right|_{x=2}$

Different ways to find the value of: $\left. \frac{dy}{dx} \right|_{x=2}$

I. Given an equation: $y = 3x^2 + 2x$

II. Given a graph:



III. Given a TOV:

| | | | | | | |
|-----|---|----|----|----|----|----|
| x | 0 | 1 | 2 | 3 | 4 | 5 |
| y | 6 | 16 | 30 | 48 | 70 | 96 |

Using the TOV below, determine the values of: $\left. \frac{dy}{dx} \right|_{x=4}$, $\left. \frac{dy}{dt} \right|_{t=4}$, $\left. \frac{dy}{ds} \right|_{s=4}$

| | | | | | | |
|-----|----|-----|----|-----|----|-----|
| Y | 10 | 14 | 16 | 20 | 26 | 36 |
| x | 2 | 3 | 4 | 5 | 6 | 7 |
| t | 3 | 3.5 | 4 | 4.5 | 5 | 5.5 |
| s | 2 | 4 | 6 | 8 | 10 | 12 |

What is a **Related Rates** word problem?

Example 1

One face of a plant cell has a rectangular shape with an initial length of $3\mu m$ and width of $2\mu m$. As the cell absorbs water, the length of the face increases at a rate of $1\mu m$ per second and the width increases at a rate of $2\mu m$ per second. Determine how fast the area of the face of the cell is changing after 2 seconds.

A. Numeric Method

B. Algebraic Method

Diagram

Given

Find

Relationship/Equation/Formula

Differentiate with respect to t

Substitute values to determine answer

Example 2

A 10 foot ladder is leaning against a brick wall. The top of the ladder is originally 8.5 feet above the ground. The top of the ladder begins to fall at a constant rate of 0.5 feet per second. At what rate is the foot of the ladder moving when the top of the ladder is 5.5 feet from the ground?

Example 3

Train A is travelling west towards Vancouver at 120mph, while train B is travelling north, away from Vancouver, at 90mph. At time, $t = 0$, train A is 10 miles east of Vancouver, and train B is 20 miles north of Vancouver. Calculate the rate at which the distance between the trains is changing at time, $t = 10\text{min}$.

Example 4

A spy is tracking a rocket launch 10km away, through a telescope to determine its velocity. At a certain moment, the angle between the ground and the spy's telescope is $\frac{\pi}{3}$ radians and is increasing at a rate of 0.5 radians/min. What is the rocket's velocity at that moment?