

# Calculus 12

## 1.3 - Continuity

A function  $f(x)$  is **continuous** on the interval  $x \in (a, b)$  if  $f(x)$  is defined for all  $x \in (a, b)$ .

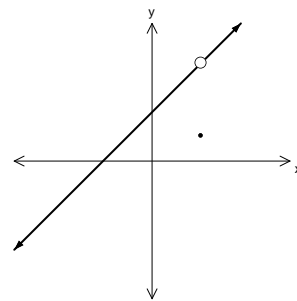
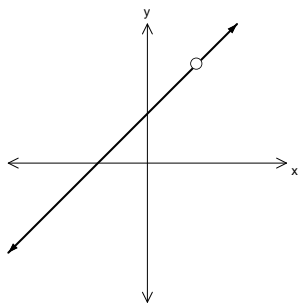
A function  $f(x)$  is **discontinuous** at  $x = c$  if  $f(c) = u/d \rightarrow$  at  $x = c$  the graph of  $f(x)$  has a "break."

Every function is continuous over its domain.

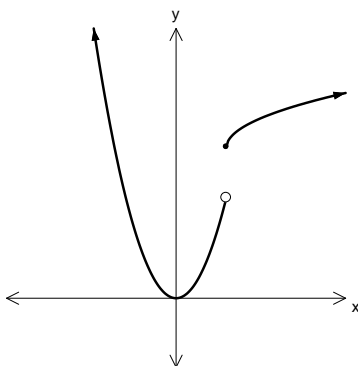
The following functions are continuous for  $x \in \mathbb{R}$  : Linear, Polynomial, Absolute Value, Sin, Cos, Exponential.

### Types of Discontinuities

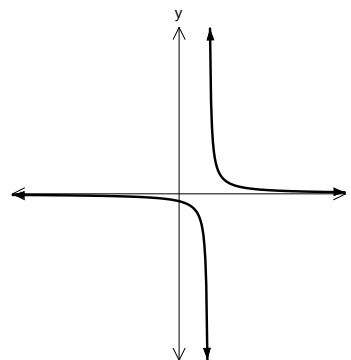
#### Point Discontinuity or Hole



#### Jump Discontinuity



#### Asymptotic/Infinite Discontinuity



## The Continuity Theorem

$f(x)$  is continuous at  $x = c$  if:

I.

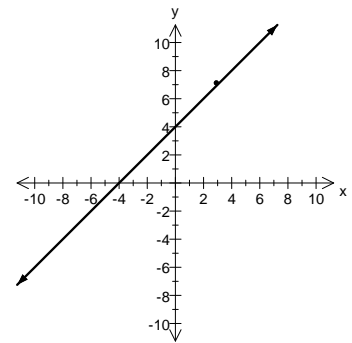
II.

III.

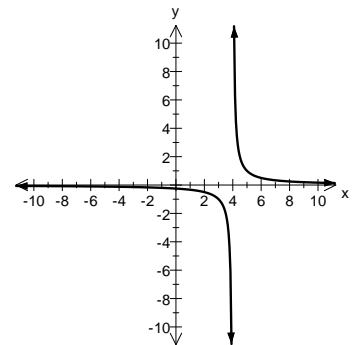
→ If  $f(x)$  is continuous at  $x = c$  then \_\_\_\_\_

### Examples

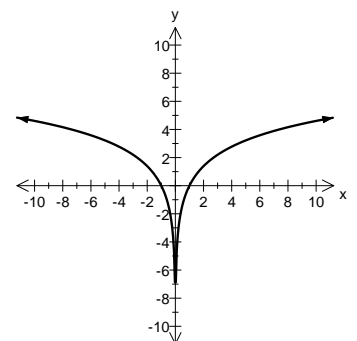
1.  $\lim_{x \rightarrow 3} x + 4$



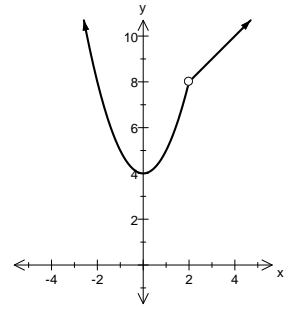
2.  $\lim_{x \rightarrow 2} \frac{1}{x-4}$



3.  $\lim_{x \rightarrow e} \ln(x^2)$



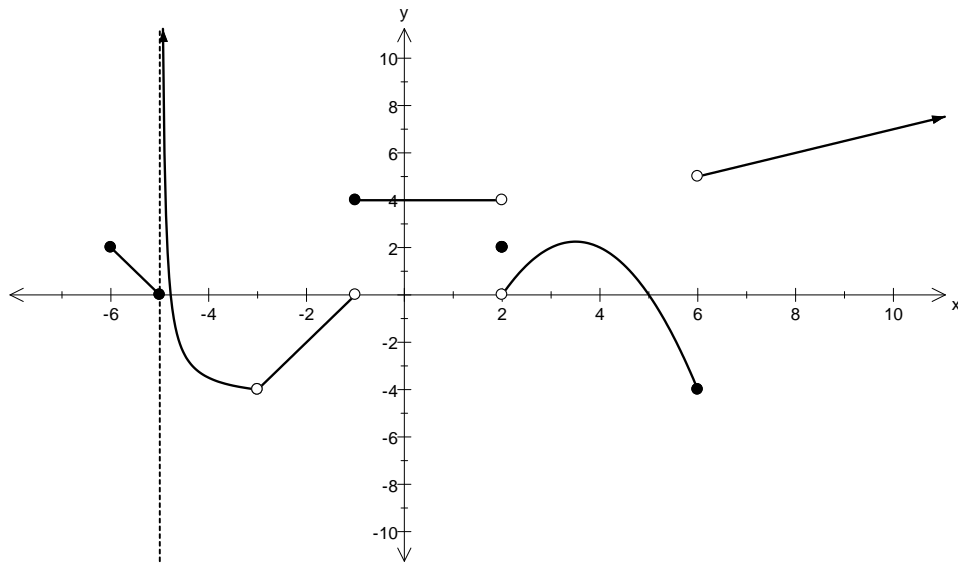
4.  $f(x) = \begin{cases} x^2 + 4 & x \leq 2 \\ x + 6 & x > 2 \end{cases}$  Is  $f(x)$  continuous at  $x = 2$  ?



5.  $f(x) = \begin{cases} 2 \sin x & x \leq \frac{3\pi}{2} \\ -1.5x + 5 & x > \frac{3\pi}{2} \end{cases}$  Is  $f(x)$  continuous at  $x = \frac{3\pi}{2}$  ?

6.  $f(x) = \begin{cases} x^2 - m & x < e \\ \ln(x) & x \geq e \end{cases}$  Determine the value of  $m$  that makes  $f(x)$  continuous for  $x \in \mathbb{R}$ .

## Intervals of Continuity



State the intervals of continuity for the function given above.

At the following values of  $x$ , state whether the function is *continuous from the left*, *continuous from the right*, *continuous from both left and right*, or *neither*.

$x = -6$  :

$x = 2$  :

$x = -5$  :

$x = 5$  :

$x = -3$  :

$x = 6$  :

$x = -1$  :

$x = 8$  :

$x = 0$  :