September-11-12
12:14 PM

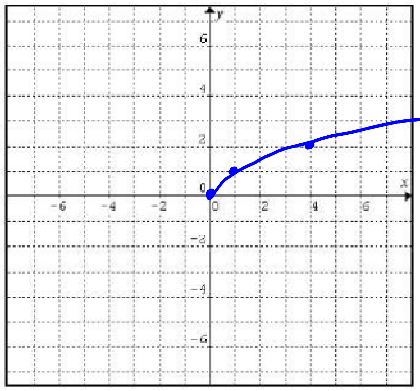
Transformations

1.2 - Reflections

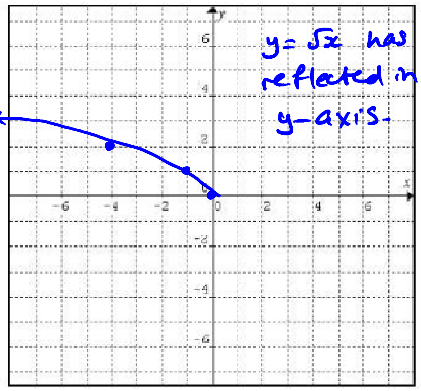
$x \to -x$ x has been mult. by −1.

Given graph of $y = f(x)$, sketch the graph of $y = f(-x)$.

$y = \sqrt{x}$

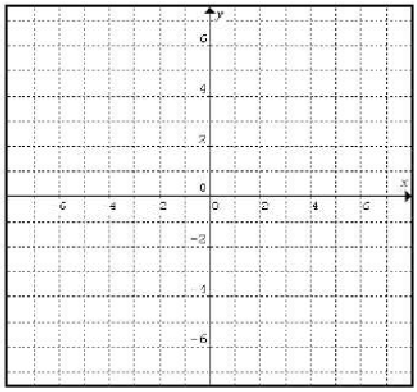


$y = \sqrt{(-x)}$

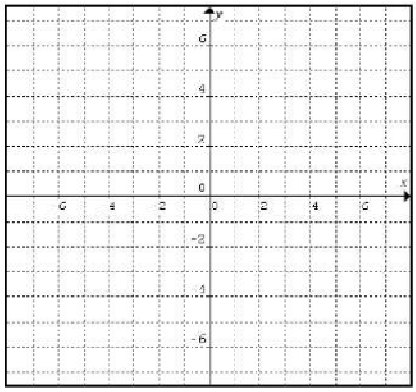


y = √x has been reflected in the y-axis.

$y = \left(x^3 - 125\right)^{\frac{1}{3}}$



$y = \left((-x)^3 - 125\right)^{\frac{1}{3}}$



Given $y = \sqrt{x}$, if we replace (x) with $(-x)$ we get $y = \sqrt{(-x)}$, this transforms the graph of $y = \sqrt{x}$ by:

Reflection in the y axis. All original points have their x-coords mult. by −1.

Given $y = \left(x^3 - 125\right)^{\frac{1}{3}}$, if we replace (x) with $(-x)$ we get $y = \left((-x)^3 - 125\right)^{\frac{1}{3}}$, this transforms the graph of $y = \sqrt{x}$ by:

$x \to -x$

In general: Given, $y = f(x)$. If (x) is replaced with $(-x)$, the new equation is: $y = f(-x)$

This causes the graph of $y = f(x)$ to reflect in the y-axis

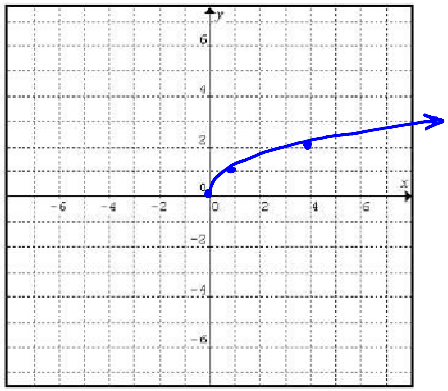
If (p,q) is a point on $y = f(x)$, then $(-p, q)$ will be a point on $y = f(-x)$

original

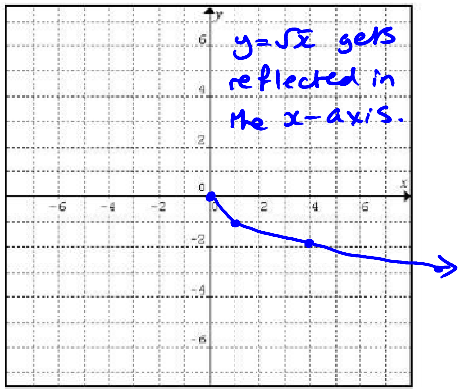
$y \to -y$

Given graph of $y = f(x)$, sketch the graph of $y = -f(x)$ or $(-y) = f(x)$

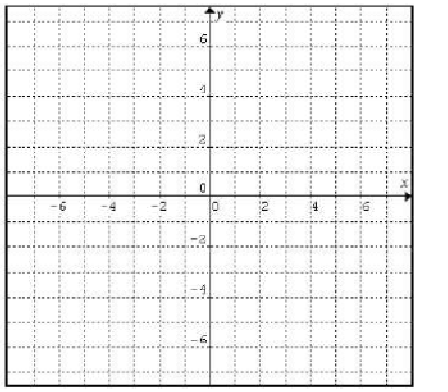
$y = \sqrt{x}$



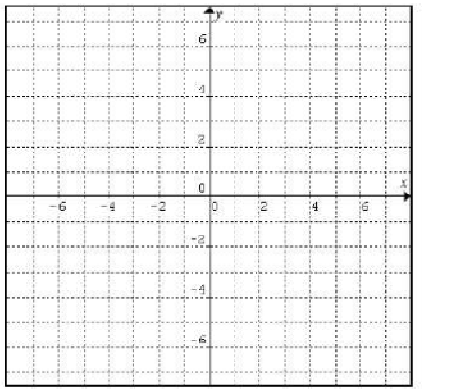
$y = -\sqrt{x}$ or $(-y) = \sqrt{x}$



$y = -(25 - x^2)^{\frac{1}{2}}$



$y = -\left[(25 - x^2)^{\frac{1}{2}}\right]$ or $(-y) = (25 - x^2)^{\frac{1}{2}}$



Given $y = \sqrt{x}$, if we replace (y) with $(-y)$ we get $y = -\sqrt{x}$ or $(-y) = \sqrt{x}$, this transforms the graph of $y = \sqrt{x}$ by:

Reflection in the x-axis.

All original y-coords get mult. by -1.

Given $y=(25-x^2)^{\frac{1}{2}}$, if we replace (y) with $(-y)$ we get $y=-\left[(25-x^2)^{\frac{1}{2}}\right]$ *or* $(-y)=(25-x^2)^{\frac{1}{2}}$, this transforms the graph of $y=(25-x^2)^{\frac{1}{2}}$ by:

In general: Given, $y=f(x)$. If (y) is replaced with $(-y)$, the new equation is: $-y=f(x)$ or $y=-f(x)$

This causes the graph of $y=f(x)$ to reflect in the x-axis

If (p,q) is a point on $y=f(x)$, then $(p,-q)$ will be a point on $y=-f(x)$

Invariant Points

Invariant Points are all points on $y=f(x)$ that are **not** changed by the transformation.

Transformation $y=f(-x)$ has *Invariant Points* on the y-axis where the x-coords = 0

Transformation $y=-f(x)$ has *Invariant Points* on the x-axis where the y-coords = 0

Examples

1. The graph of $y=2x^2+3x-1$ is reflected in the y axis. Determine the equation of the transformed graph.

$x \to -x$

$y=2(-x)^2+3(-x)-1 = 2x^2-3x-1$

2. The graph of $y=\sqrt{x-1}+2$ is transformed to $y=-f(x)$. Determine the equation of the new graph.

$y=f(x)$ $\quad (-y)=f(x)$

$-y=\sqrt{x-1}+2 \Rightarrow y=-(\sqrt{x-1}+2)=-\sqrt{x-1}-2$

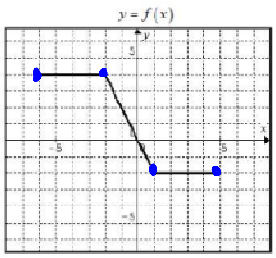
3. Given $(3,-1)$ is a point on $y=f(x)$, find the coordinates of a point on $-y=f(-x)$.

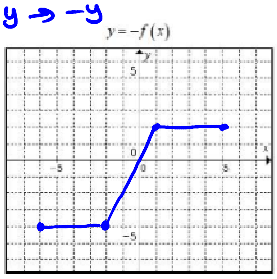
$(3,-1) \to (-3,-1) \to (-3,+1)$

$x \to -x$
$y \to -y$

4. Sketch the graphs after the given transformations, below.

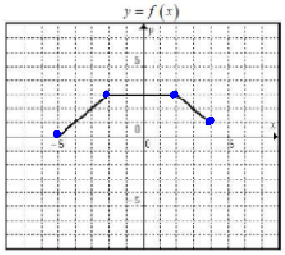
$y \to -y$

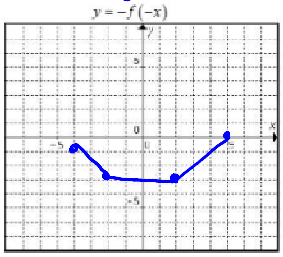




$(-6,4) \to (-6,-4)$
$(-2,4) \to (-2,-4)$
$(1,-2) \to (1,2)$
$(5,-2) \to (5,2)$

refl in x-axis
y-axis





$(-5,0) \to (5,0)$
$(-2,3) \to (2,-3)$
$(2,3) \to (-2,-3)$
$(4,1) \to (-4,-1)$

The Inverse Transformation.

The Inverse Transformation.

What is an *Inverse Function?*

A function that UNDOES the operations done by another function.

How to determine the Inverse of a given function, Algebraically.

1. Swap x and y
2. Isolate the y.

Example 1. Determine the inverse of $y=2x+4$ algebraically and sketch the graphs of both functions.

$y = 2x+4$. Find its inverse.

$x = 2y+4$

Isolate the y:

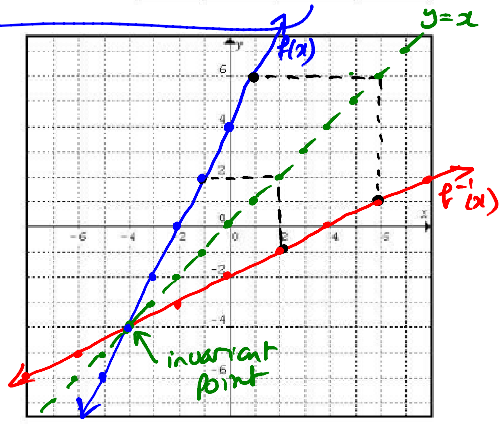
$x-4 = 2y$

$\frac{x-4}{2} = y \Rightarrow$ Inverse function.

written as: $f^{-1}(x) = \frac{x-4}{2} = \frac{1}{2}x - 2$

$f(x) = 2x+4$

$y=f(x)$		$y=f^{-1}(x)$	
x	y	x	y
0	4	4	0
1	6	6	1



Example 2. Given, $f(x)=x^2-4$, determine $y=f^{-1}(x)$ *or* $x=f(y)$. Graph $y=f(x)$ and its inverse on your GC.

$y=f(x)$		$y=f^{-1}(x)$	
x	y	x	y
2		0	
1		-3	

this is NOT "negative 1"

Given $y=f(x)$ and its inverse $y=f^{-1}(x)$ *or* $x=f(y)$:

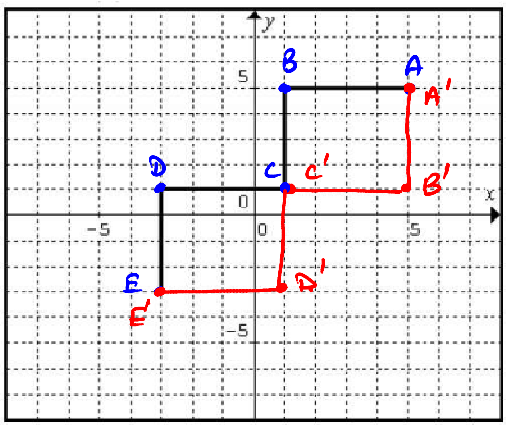
This is NOT "negative 1"

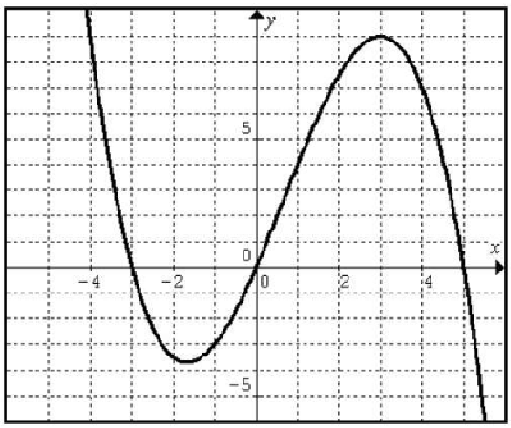
Given, $y = f(x)$ and its inverse, $y = f^{-1}(x)$ or $x = f(y)$:

- If point (p, q) is on the graph of $y = f(x)$ then the point (q, p) will be on the graph of $y = f^{-1}(x)$.
- The graph of $y = f^{-1}(x)$ can be obtained from the graph of $y = f(x)$ by reflecting it in: the line $y = x$.
- The *Invariant Points* for the Inverse Transformation are found on the line $y = x$.

WARNING!!!! $y = f^{-1}(x)$ is NOT the same as $\frac{1}{f(x)} \rightarrow [f(x)]^{-1}$

Given the graphs of $y = f(x)$ below, sketch the graphs of $y = f^{-1}(x)$.





Challenge Question

The transformation $y = f(x - 3) + 2$ is applied to the graph of $y = f(x)$ and then it is transformed by $y = f(-x)$.

3 rt and 2 up

refl. in y-axis

a. If the point $(-2, 7)$ was on the graph of $y = f(x)$, determine the coordinates of the point on the new graph at the end of all transformations.

$(-2, 7) \rightarrow (1, 9) \rightarrow (-1, 9)$

b. If the point $(5, -8)$ is a point on the new graph at the end of all transformations, determine the coordinates of the point on the original graph, $y = f(x)$.

$_____ \leftarrow (-5, -8) \leftarrow (5, -8)$

p. 12 : #1 - 9

p. 28: #1

p. 51: #1-3, 5-9, 20*

Quiz tomorrow

① codes

② apply to eqns.

Find the inverse of: $y = \frac{x}{x+1}$

$$x = \frac{y}{y+1}$$

$$xy + x = y$$

$$xy - y = -x$$

$$y(x-1) = -x$$

$$y = \frac{-x}{x-1} = \frac{x}{1-x}$$