

# Calculus 12

## 1.1 - Introduction to Calculus

Calculus is the study of:

1. **Limits** - Determining the value of  $f(x)$  as the value of  $x$  approaches a specified value.
2. **Derivatives** - A measure of the rate of change of a function and its application to real-world problems.
3. **Integrals** - A measure of the net accumulation of area under the graph of  $f(x)$  over a given interval of  $x$  and its application to real-world problems.

### Rate of Change of a Function

The rate of change of a function is a measure of:

$$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a} = \frac{f(x) - f(x_0)}{x - x_0} = \frac{dy}{dx}$$

e.g.

**Secant Line** A straight line segment between any 2 points.

There are 2 types of ROC:

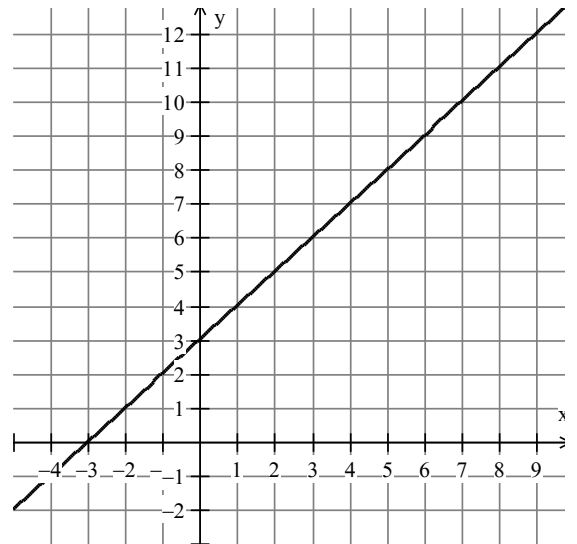
**Average ROC:** Change in  $y$  over a **FINITE** change in  $x$  from  $x = a$  to  $x = b$ .

Is the slope of the Secant Line with endpoints at  $x = a$  and  $x = b$ .

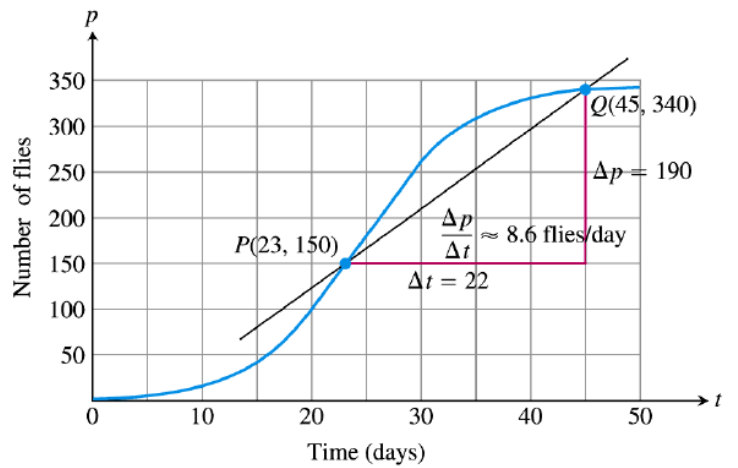
**Instantaneous ROC:** Change in  $y$  over an **INFINITESIMALLY** small change in  $x$  **AT**  $x = a$ .

Is the slope of the Tangent Line at  $x = a$ .

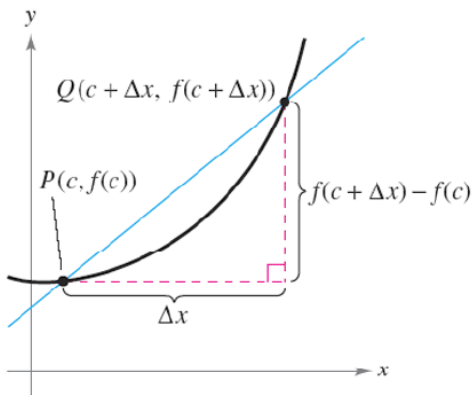
**How to find Average and Instantaneous ROC**  
**Linear Relations**



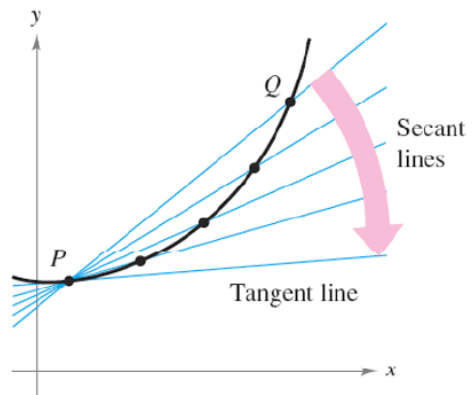
**Non-Linear Relations**



**The Tangent Line Problem & the reason why we need to study limits!**



(a) The secant line through  $(c, f(c))$  and  $(c + \Delta x, f(c + \Delta x))$



(b) As  $Q$  approaches  $P$ , the secant lines approach the tangent line.